Error correction codes

Block Error Correction Codes

- Error detection requires blocks to be retransmitted.
- This is inadequate for wireless communication for two reasons.
- The bit error rate on a wireless link can be quite high, which would result in a large number of retransmissions.
- In some cases, especially satellite links, the propagation delay is very long compared to the transmission time of a single frame.
- The result is a very inefficient system.
- It is desirable to be able to correct errors without requiring retransmission. Using the bits that were transmitted.

Hamming code

- The number of redundant bits can be calculated using the following formula:
- **$2^r > m + r + 1$** where, r = redundant bit, m = data bit
- Suppose the number of data bits is 7, then the number of redundant bits can be calculated using:
- $2^4 > 7 + 4 + 1$
- Thus, the number of redundant bits= 4

- Example

Hamming code=(8,4)

8 bit data block=00111001

(a) Transmitted block

Bit Position	12	11	10	9	8	7	6	5	4	3	2	1
Position Number	1100	1011	1010	1001	1000	0111	0110	0101	0100	0011	0010	0001
Data Bit	D8	D7	D6	D5		D4	D3	D2		D1		
Check Bit					C8				C4		C2	C1
Transmitted Block	0	0	1	1	0	1	0	0	1	1	1	1
Codes			1010	1001		0111				0011		

(b) Check bit calculation prior to transmission

Position	Code
10	1010
9	1001
7	0111
3	0011
XOR = C8 C4 C2 C1	0111

(c) Received block

Bit Position	12	11	10	9	8	7	6	5	4	3	2	1
Position Number	1100	1011	1010	1001	1000	0111	0110	0101	0100	0011	0010	0001
Data Bit	D8	D7	D6	D5		D4	D3	D2		D1		
Check Bit					C8				C4		C2	C1
Received Block	0	0	1	1	0	1	1	0	1	1	1	1
Codes			1010	1001		0111	0110			0011		

(d) Check bit calculation after reception

Position	Code
Hamming	0111
10	1010
9	1001
7	0111
6	0110
3	0011
XOR = syndrome	0110

BCH

- Bose, Chaudhuri, and Hocquenghem (BCH) codes are among the most powerful cyclic block codes and are widely used in wireless applications.
- For any positive pair of integers m and t, there is a binary (n, k) BCH code with the following parameters:
- Block length: $n = 2^m - 1$
- Number of check bits: $n - k \le mt$
- Minimum distance: $d_{min} \ge 2t + 1$

- This code can correct all combinations of t or fewer errors.

Reed solomon codes

- Reed-Solomon (RS) codes are a widely used subclass of nonbinary BCH codes.
- With RS codes, data are processed in chunks of m bits, called symbols.
- An (n, k) RScode has the following parameters:
- Symbol length:m bits per symbol
- Block length: $n = 2^m - 1$ symbols
- Data length: k symbols
- Size of check code: $n - k = 2t$ symbols $=m(2t)$ symbols
- Minimum distance:$d_{min} = 2t + 1$ symbols

Let $t = 1$ and $m = 2$. Denoting the symbols as 0, 1, 2, 3 we can write their binary equivalents as $0 = 00; 1 = 01; 2 = 10; 3 = 11$. The code has the following parameters.

$$n = 2^2 - 1 = 3 \text{ symbols} = 6 \text{ bits}$$
$$(n - k) = 2 \text{ symbols} = 4 \text{ bits}$$

This code can correct any burst error that spans a symbol of 2 bits.

Application and example

- Storage devices (including tape, Compact Disk, DVD, barcodes, etc)
- Wireless or mobile communications (including cellular telephones, microwave links, etc)
- Satellite communications
- Digital television / DVB
- High-speed modems such as ADSL, xDSL, etc.

- A popular Reed-Solomon code is RS(255,223) with 8-bit symbols. Each codeword contains 255 code word bytes, of which 223 bytes are data and 32 bytes are parity. For this code:
- $n = 255$, $k = 223$, $s = 8$
- $2t = 32$, $t = 16$
- The decoder can correct any 16 symbol errors in the code word: i.e. errors in up to 16 bytes anywhere in the codeword can be automatically corrected.
- Given a symbol size s, the maximum codeword length (n) for a Reed-Solomon code is $n = 2s – 1$
- For example, the maximum length of a code with 8-bit symbols (s=8) is 255 bytes.

Atmospheric Absorption

- At frequencies above 10 GHz, radio waves propagating through the atmosphere are subject to molecular absorption.
- The absorption as a function of frequency is very uneven
- There is a peak of water vapor absorption at around 22 GHz and a peak of oxygen absorption near 60 GHz.

- Vegetation and rain too effect the waves.