Module - 6

Testing of Hypothesis

* **Statistical Hypothesis:-**
A statistical hypothesis is some statement or assertion about a population or an equivalently about the probability distribution characterising a population, which we want to verify on the basis of information available from a sample.

If the statistical hypothesis specifies the population completely then it is termed as a simple statistical hypothesis; otherwise it is called a composite statistical hypothesis.

* **Hypothesis Testing:-** A test of a statistical hypothesis is a two-action decision problem after the experimental sample values have been obtained, the two-actions being the acceptance or rejection of the hypothesis under the consideration.

① **Null Hypothesis:-** The neutral or non-committal attitude of the statistician or decision-maker before the sample observations are taken is the keynote of the null hypothesis.

The null hypothesis is denoted by H_0.

② **Alternate Hypothesis:—** It is desirable to state that what is called an alternative hypothesis in respect of every statistical hypothesis being tested because the acceptance or rejection of null hypothesis is meaningful only when it is being tested against a rival hypothesis which should rather be explicitly mentioned. Alternative hypothesis is usually denoted by H_1.

* **Two Types of Errors:—**

The decision to accept or reject null hypothesis H_0 is made on the basis of the information supplied by the observed sample observations. The conclusion drawn on the basis of a particular sample may not always be true in respect of the population. The four possible situations that arise in any test procedure are given by —

		Decision from sample	
		Reject H_0	Accept H_0
True state	H_0 true	Wrong (Type I Error)	correct
	H_0 False (H_1 true)	correct	Wrong (Type II Error)

① The error of rejecting Ho (accepting H_1) when Ho is true is called Type I error and the error of accepting Ho when Ho is false (H_1 is true) is called type II error. The probabilities of type I and type II errors are denoted by α and β respectively. Thus,

α = probability of type I error
= probability of rejecting Ho when Ho is true

β = probability of type II error
= probability of accepting Ho when Ho is false.

* <u>Level of significance:-</u>
α, the probability of type 1 error, is known as the level of significance of the test.

* <u>Note:-</u>
① For single sample, if size is n then degree of freedom (d.f.) = n-1
② For two samples with size m and n
degree of freedom (d.f.) = ~~(m-1)(n-1)~~
= (m+n-2)

* Students "t" distribution :- (t-test) :-

The t-distribution has a wide number of applications in statistics, some of which are

(i) To test if the sample mean ($\bar{x}$) differs significantly from the hypothetical value μ of the population mean.

(ii) To test the significance of the difference between two sample means.

(iii) To test the significance of an observed sample correlation coefficient and sample regression coefficient.

(iv) To test the significance of observed partial correlation coefficient.

* t-Test for single mean :-

Suppose we want to test -

(i) If a random sample x_i ($i = 1, 2, 3, ---, n$) of size n has been drawn from a normal population with a specified mean, say μ_0.

OR

(ii) If the sample mean differs significantly from the hypothetical value μ_0 of the population mean.

Under the null hypothesis, H_0:

i) The sample has been drawn from the population with mean μ_0 or

ii) there is no significant difference betⁿ the sample mean $\bar{x}$ and the population mean μ_0,

the statistic $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$, where $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$,

follows student's t= distribution with $(n-1)$ d.f.

We now compare the calculated value of t with the tabulated value at certain level of significance. If calculated $|t| >$ tabulated t, null hypothesis is rejected and if calculated $|t| <$ tabulated t, H_0 may be accepted at the level of significance adopted.

Que] The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising compaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. was the advertising compaign successful?

[Tabulated value of t for 21 d.f. at 5% level of significance is 1.72]

Solution:- We are given that,

Number of stores = 22 $\Rightarrow n = 22$

Also, the weekly sales in 22 stores for a typical week increased to 153.7

$\therefore \overline{x} = 153.7$

and standard deviation $(s) = 17.2$

Null hypothesis: The advertising compaign is not successful

$\therefore \quad H_0 : \mu = 146.3$

Alternative hypothesis (H_1): $\mu > 146.3$

under H_0, the test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{153.7 - 146.3}{17.2/\sqrt{22}} = \frac{7.4}{17.2/4.69}$$

$$t = \frac{7.4}{3.67} = 2.02$$

$\therefore$ calculate $t = 2.02 >$ tabulated $t = 1.72$

$\therefore$ we reject the null hypothesis H_0 and hence accept alternative hypothesis H_1.

$\therefore$ we conclude that $\mu > 146.3$ i.e. the advertising compaign was definitely successful in promoting sales.

Que] A random sample of 10 boys had the following I.Q's.

70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of a population mean I.Q. of 100? Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie.

[Tabulated $t_{0.05}$ for $(10-1) = 9$ d.f. is 2.262]

Solution:-

Define H_0 and H_1 as -

Null Hypothesis (H_0): The data are consistent with the assumption of a mean I.Q. of 100 in the population. i.e. $\mu = 100$.

Alternative Hypothesis (H_1):- $\mu \neq 100$

under H_0, test statistic is

$$t = \frac{(\bar{x} - \mu)}{s/\sqrt{n}} \sim t_{(n-1)}$$

Where $\bar{x}$ and s are computed from the sample value of I.Q.'s

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

x	$x-\bar{x}$	$(x-\bar{x})^2$
70	$70-97.2=-27.2$	739.84
120	$120-97.2=22.8$	519.84
110	$110-97.2=12.8$	163.84
101	$101-97.2=3.8$	14.44
88	$88-97.2=-9.2$	84.64
83	$83-97.2=-14.2$	201.64
95	$95-97.2=-2.2$	4.84
98	$98-97.2=0.8$	0.64
107	$107-97.2=9.8$	96.04
100	$100-97.2=2.8$	7.84
$\Sigma x = 972$		$\Sigma(x-\bar{x})^2 = 1833.60$

Here $n=10$ and $\Sigma x = 972$

$$\therefore \bar{x} = \frac{1}{n}\sum_{i=1}^{10} x_i = \frac{1}{10}\times 972$$

$$\therefore \boxed{\bar{x} = 97.2}$$

$$\therefore s^2 = \frac{1}{n-1}\sum_{i=1}^{10}(x-\bar{x})^2 = \frac{1}{10-1}\times 1833.60$$

$$\therefore s^2 = \frac{1}{9}\times 1833.60$$

$$\therefore \boxed{s^2 = 203.73} \Rightarrow \boxed{s = 14.27}$$

$$\therefore t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{97.2-100}{14.27/\sqrt{10}}$$

$$t = \frac{-2.8}{14.27/3.16} = \frac{-2.8}{4.51} = -0.6208$$

$$\therefore |t| = |-0.6208| = 0.6208$$

$\therefore$ calculated $t = 0.6208$ < tabulated $t = 2.262$

$\therefore$ we accept null hypothesis H_0 and we may conclude that the data are consistent with the assumption of mean I.Q. of 100 in the population.

Que] The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. It is reasenable to believe that the average height is greater than 64 inches ? Test at 5% significance level assuming that for 9 degree of freedom $P(t > 1.83) = 0.05$

solution:- Let define the following hypothesis

Null hypothesis (H_0) : $\mu = 64$ inches

Alternate hypothesis (H_1) : $\mu > 64$ inches

Calculation for sample mean and S.D. is given as

x	70	67	62	68	61	68	70	64	64	66	Total $= 660$
$x-\bar{x}$	4	1	-4	2	-5	2	4	-2	-2	0	$\Sigma x-\bar{x} = 0$
$(x-\bar{x})^2$	16	1	16	4	25	4	16	4	4	0	$\Sigma(x-\bar{x})^2 = 90$

Here $n = 10$

$\Sigma x = 660, \quad \Sigma(x-\bar{x})^2 = 90$

$\therefore \bar{x} = \frac{1}{n}\Sigma x = \frac{1}{10} \times 660 = 66 \Rightarrow \boxed{\bar{x} = 66}$

$$\therefore \quad s^2 = \frac{1}{n-1} \Sigma (x-\bar{x})^2$$

$$\therefore \quad s^2 = \frac{1}{10-1} \times 90 = \frac{1}{9} \times 90 = 10$$

$$\therefore \quad s = \sqrt{10}$$

$$\therefore \quad \boxed{s = 3.16}$$

$\therefore$ under Ho, the test statistics is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{66-64}{3.16/\sqrt{10}} = \frac{2}{3.16/3.16}$$

$$\therefore \quad \boxed{t = 2}$$

$\therefore$ calculated $t = 2 > $ tabulated $t = 1.83$

$\therefore$ we ~~accept~~ reject null hypothesis Ho.

$\therefore$ we conclude that, average height is greater than 64 inches.

* t- Test for Difference of means :-

Suppose we want to test if two independent samples x_i $(i = 1, 2, ---, n_1)$ and y_j $(j = 1, 2, ---, n_2)$ of sizes n_1 and n_2 have been drawn from two normal populations with means μ_x and μ_y respectively.

Under the null hypothesis (H_0) that the samples haven been drawn from the normal populations with means μ_x and μ_y and under the assumptions that the population variance are equal i.e.

$$\sigma_x^2 = \sigma_y^2 = \sigma^2$$

The statistics is

$$t = \frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{s\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where $\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$

$$\bar{y} = \frac{1}{n_2} \sum_{j=1}^{n_2} y_i$$

$$s^2 = \frac{1}{n_1 + n_2 - 2} \left[\sum_{i=1}^{n_1} (x_i - \bar{x})^2 + \sum_{j=1}^{n_2} (y_j - \bar{y})^2 \right]$$

Que] Below are given the gain in weights (in kgs) of animals on two diets A and B.

	Gain in weight (in kgs)														
Diet A	25	32	30	34	24	14	32	24	30	31	35	25	—	—	—
Diet B	44	34	22	10	47	31	40	30	32	35	18	21	35	29	22

test, if the two diets differ significantly as regards their effects on increase in weight.

[Tabulated $t_{0.05}$ for $(12+15-2) = 25$ d.o.f. is 2.06]

Solution:- Let define following hypothesis,

Null hypothesis (H_0): $\mu_x = \mu_y$, that is no significant difference between the mean increase in weight due to diets A and B.

Alternative hypothesis (H_1):- $\mu_x \neq \mu_y$

The calculation of sample mean and S.D. is given

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{j=1}^{n_2} y_j$$

$$s^2 = \frac{1}{n_1+n_2-2} \left[\sum_{i=1}^{n_1} (x_i - \bar{x})^2 + \sum_{j=1}^{n_2} (y_j - \bar{y})^2 \right]$$

Diet A			Diet B		
x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	y_i	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
25	$25-28=-3$	9	44	$44-30=14$	196
32	$32-28=4$	16	34	$34-30=4$	16
30	$30-28=2$	4	22	$22-30=-8$	64
34	$34-28=6$	36	10	$10-30=-20$	400
24	$24-28=-4$	16	47	$47-30=17$	289
14	$14-28=-14$	196	31	$31-30=1$	1
32	$32-28=4$	16	40	$40-30=10$	100
24	$24-28=-4$	16	30	$30-30=0$	0
30	$30-28=2$	4	32	$32-30=2$	4
31	$31-28=3$	9	35	$35-30=5$	25
35	$35-28=7$	49	18	$18-30=-12$	144
25	$25-28=-3$	9	21	$21-30=-9$	81
			35	$35-30=5$	25
			29	$29-30=-1$	1
			22	$22-30=-8$	64
$\Sigma x_i = 336$	$\Sigma(x_i - \bar{x}) = 0$	$\Sigma(x_i - \bar{x})^2 = 380$	$\Sigma y_i = 450$	$\Sigma(y_i - \bar{y}) = 0$	$\Sigma(y_i - \bar{y})^2 = 1410$

Here, $n_1 = 12$ and $n_2 = 15$

From above table,

$\Sigma x_i = 336 \Rightarrow \bar{x} = \frac{1}{n_1} \Sigma x_i = \frac{1}{12} \times 336 = 28$

$\Sigma y_i = 450 \Rightarrow \bar{y} = \frac{1}{n_2} \Sigma y_i = \frac{1}{15} \times 450 = 30$

Also, from above table,

$\Sigma(x_i - \bar{x})^2 = 380$ and $\Sigma(y_i - \bar{y})^2 = 1410$

$$\therefore s^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2\right]$$

$$\therefore s^2 = \frac{1}{12+15-2}\left[380+1410\right]$$

$$s^2 = \frac{1}{25} \times 1790$$

$$\therefore s^2 = 71.6$$

$$\therefore s = \sqrt{71.6}$$

$$\therefore \boxed{s = 8.46}$$

$\therefore$ under null hypothesis (Ho),

$$t = \frac{\bar{x}-\bar{y}}{s\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{28-30}{8.46\sqrt{\frac{1}{12}+\frac{1}{15}}}$$

$$t = \frac{-2}{8.46\sqrt{15+12/180}} = \frac{-2}{8.46\sqrt{27/180}}$$

$$t = \frac{-2}{8.46\times\sqrt{0.15}} = \frac{-2}{8.46\times 0.39}$$

$$t = \frac{-2}{3.2994} = -0.6061$$

$$\therefore |t| = |-0.6061| = 0.6061$$

$\therefore$ calculated $t = 0.6061 <$ tabulated $t = 2.06$

$\therefore$ We accept null hypothesis Ho.

$\therefore$ We may conclude that the two diets do not differ significantly as regards their effect on increase in weight.

Que] Sample of two types of electric light bulbs were tested for length of life and following data were obtained.

	Type I	Type II
Sample No.	$n_1 = 8$	$n_2 = 7$
Sample means	$\bar{x}_1 = 1234$ Hrs	$\bar{x}_2 = 1036$ Hrs
Sample S.D.	$s_1 = 36$ Hrs	$s_2 = 40$ Hrs

If the difference in the mean sufficient to warrant that type I is superior to type II regarding length of life.

Answer:-

Here $n_1 = 8$, $n_2 = 7$

$\bar{x}_1 = 1234$, $\bar{x}_2 = 1036$

$s_1 = 36$, $s_2 = 40$

$\therefore$ Degree of freedom (d.f.) $= (n_1 + n_2 - 2)$

$= 8 + 7 - 2 = 13$

The level of significance is not given, we consider it as $5\% = 0.005$

$\therefore$ From table, tabulated value of t for 13 d.f. at 5% level of significance is 2.16

Now, define the hypothesis.

Null hypothesis (H0):- $\mu x = \mu y$ that is, the two types I and II of electric bulbs are identical.

Alternative hypothesis (H_1) :- $\mu_x > \mu_y$ that is type I is superior to type II.

under H_0, the test statistics is -

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $s^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2\right]$

$$\therefore s^2 = \frac{1}{n_1+n_2-2}\left[n_1 s_1^2 + n_2 s_2^2\right]$$

$$\therefore s^2 = \frac{1}{8+7-2}\left[8\times(36)^2 + 7\times(40)^2\right]$$

$$\therefore s^2 = \frac{1}{13}\left[8\times 1,269 + 7\times 1600\right]$$

$$s^2 = \frac{1}{13}\left[10,152 + 11,200\right]$$

$$s^2 = \frac{1}{13}\times 21,352 = 1,642.46$$

$$\therefore s = \sqrt{1642.46} = 40.53$$

$$\therefore \boxed{s = 40.53}$$

$$\therefore t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\therefore \quad t = \frac{1234 - 1036}{40.50\sqrt{\frac{1}{8}+\frac{1}{7}}}$$

$$\therefore t = \frac{198}{40.50\sqrt{\frac{7+8}{56}}} = \frac{198}{40.50 \times \sqrt{15/56}}$$

$$t = \frac{198}{40.50 \times \sqrt{0.2678}} = \frac{198}{40.50 \times 0.5175}$$

$$t = \frac{198}{20.96} = 9.45$$

$$\therefore \boxed{t = 9.45}$$

$\therefore$ calculated $t = 9.45 >$ tabulated $t = 1.77$

$\therefore$ we reject null hypothesis H_0.

$\therefore$ Hence two types of electric bulbs differ significantly.

Que] H.W. The heights of six randomly chosen sailers are (in inches): 63, 65, 68, 69, 71 and 72. Those of 10 randomly choosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss, the light that these data throw on the suggestion that sailors are on the average taller than soldiers.

Example 16·13. *To test the claim that the resistance of electric wire can be reduced by at least 0·05 ohm by alloying, 25 values obtained for each alloyed wire and standard wire produced the following results :*

	Mean	*Standard deviation*
Alloyed wire	*0·083 ohm*	*0·003 ohm*
Standard wire	*0·136 ohm*	*0·002 ohm*

Test at 5% level whether or not the claim is substantiated.

Solution. *Null Hypothesis* $H_0 : \mu_1 - \mu_2 \geq 0{\cdot}05$, [*i.e.*, the claim is substantiated]

Alternative Hypothesis $H_1 : \mu_1 - \mu_2 < 0{\cdot}05$ (Left-tailed, test)

Test Statistic. Under H_0, the test statistic is :

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0, 1)$$

where $$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{25 \times (0{\cdot}003)^2 + 25 \times (0{\cdot}002)^2}{25 + 25 - 2} = \frac{0{\cdot}000225 + 0{\cdot}0001}{48} = 0{\cdot}0000067$$

$$\therefore \quad t = \frac{(0{\cdot}083 - 0{\cdot}136) - 0{\cdot}05}{\sqrt{\left\{0{\cdot}0000067\left(\frac{1}{25} + \frac{1}{25}\right)\right\}}} = -\frac{0{\cdot}103}{0{\cdot}00071} = -145{\cdot}07$$

The (critical) tabulated value of t for 48 *d.f.*, at 5% level of significance for left-tailed test is $-1{\cdot}645$.

Conclusion. Since calculated value of t is much less than tabulated value of t, it falls in the rejection region. We, therefore, reject the null hypothesis and conclude that the claim is not substantiated.

Example 16·16. *Two laboratories carry out independent estimates of a particular chemicals in a medicine produced by a certain firm. A sample is taken from each batch, halved and the separate halves sent to the two laboratories. The following data is obtained :*

No. of samples	10
Mean value of the difference of estimates	0·6
Sum of the squares of the differences (from their means)	20

Is the difference significant ? (Value of t at 5% level for 9 d.f. is 2·262.)

Solution. Let d stand for the difference between the estimates of the chemical between the two halves of each batch, and $\bar{d}$ the mean value of the difference of estimates. In usual notations, we are given :

$$n = 10, \ \bar{d} = 0{\cdot}6, \ \Sigma(d - \bar{d})^2 = 20$$

Null hypothesis, $H_0 : \mu_1 = \mu_2$, *i.e.*, the difference is insignificant.

Alternative hypothesis, $H_1 : \mu_1 \neq \mu_2.$

Test Statistic. Under H_0, the test statistic is : $t = \dfrac{\bar{d}}{\sqrt{S^2/n}} \sim t_{10-1}$

where $\quad S^2 = \dfrac{1}{n-1}\Sigma(d - \bar{d})^2 = \dfrac{20}{9} = 2{\cdot}22 \quad \therefore \quad t = \dfrac{0{\cdot}6}{\sqrt{2{\cdot}22/10}} = \dfrac{0{\cdot}6}{0{\cdot}471} = 1{\cdot}274.$

The tabulated value of t at 5% level for 9 *d.f.*, is 2·262 (given).

Conclusion. Since calculated value of t is less than tabulated value of t, it is not significant. Hence, we may accept the null hypothesis and conclude that the difference is not significant.

TABLE I.

SIGNIFICANT VALUES $t_v(\alpha)$ of t-Distribution
(TWO-TAIL AREAS)

$P[\,|\,t\,|\, > t_v(\alpha)] = \alpha$

d.f. (v)	Probability (Level of Significance)					
	0·50	0·10	0·05	0·02	0·01	0·001
1	1·00	6·31	12·71	31·82	63·66	636·62
2	0·82	2·92	4·30	6·97	6·93	31·60
3	0·77	2·35	3·18	4·54	5·84	12·94
4	0·74	2·13	2·78	3·75	4·60	8·61
5	0·73	2·02	2·57	3·37	4·03	6·86
6	0·72	1·94	2·45	3·14	3·71	5·96
7	0·71	1·90	2·37	3·00	3·50	5·41
8	0·71	1·86	2·31	2·90	3·36	5·04
9	0·70	1·83	2·26	2·82	3·25	4·78
10	0·70	1·81	2·23	2·76	3·17	4·59
11	0·70	1·80	2·20	2·72	3·11	4·44
12	0·70	1·78	2·18	2·68	3·06	4·32
13	0·69	1·77	2·16	2·65	3·01	4·22
14	0·69	1·76	2·15	2·62	2·98	4·14
15	0·69	1·75	2·13	2·60	2·95	4·07
16	0·69	1·75	2·12	2·58	2·92	4·02
17	0·69	1·74	2·11	2·57	2·90	3·97
18	0·69	1·73	2·10	2·55	2·88	3·92
19	0·69	1·73	2·09	2·54	2·86	3·88
20	0·69	1·73	2·09	2·53	2·85	3·85
21	0·69	1·72	2·08	2·52	2·83	3·83
22	0·69	1·72	2·07	2·51	2·82	3·79
23	0·69	1·71	2·07	2·50	2·81	3·77
24	0·69	1·71	2·06	5·49	2·80	3·75
25	0·68	1·71	2·06	2·49	2·79	3·73
26	0·68	1·71	2·06	2·48	2·78	3·71
27	0·68	1·70	2·05	2·47	2·77	3·69
28	0·68	1·70	2·05	2·47	2·76	3·67
29	0·68	1·70	2·05	2·46	2·76	3·66
30	0·68	1·70	2·04	2·46	2·75	3·65
∞	0·67	1·65	1·96	2·33	2·58	3·29