8·2. DISCRETE UNIFORM DISTRIBUTION

Definition. *A r.v. X is said to have a discrete uniform distribution over the range [1, n] if its p.m.f. is expressed as follows :*

$$P(X = x) = \begin{cases} \frac{1}{n} \text{ for } x = 1, 2, \ldots, n \\ 0, \text{ otherwise} \end{cases} \quad \ldots (8{\cdot}1)$$

Here n is known as the parameter of the distribution and lies in the set of all positive integers. Equation (8·1) is also called a discrete rectangular distribution.

Such distributions can be conceived in practice if under the given experimental conditions, the different values of the random variable become equally likely. Thus for a die experiment, and for an experiment with a deck of cards such distribution is appropriate.

8·2·1. Moments. $E(X) = \frac{1}{n}\sum_{i=1}^{n} i = \frac{n+1}{2}$, $E(X^2) = \frac{1}{n}\sum_{i=1}^{n} i^2 = \frac{(n+1)(2n+1)}{6}$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{(n+1)(n-1)}{12}$$

The $m.g.f$ of X is : $M_X(t) = E(e^{tX}) = \frac{1}{n}\sum_{x=1}^{n} e^{tx} = \frac{e^t(1-e^{nt})}{n(1-e^t)}$

8·3. BERNOULLI DISTRIBUTION

Definition. *A r.v. X is said to have a Bernoulli distribution with parameter p if its p.m.f. is given by :*

$$P(X = x) = \begin{cases} p^x (1-p)^{1-x}, & \text{for } x = 0, 1 \\ 0, & \text{otherwise} \end{cases} \quad \ldots (8{\cdot}2)$$

The parameter p satisfies $0 \le p \le 1$. Often $(1 - p)$ is denoted as q.

A random experiment whose outcomes are of two types, success S and failure F, occurring with probabilities p and q respectively, is called a *Bernoulli trial*. If for this experiment, a *r.v.* X is defined such that it takes value 1 when S occurs and 0 if F occurs, then X follows a Bernoulli distribution.

8·4. BINOMIAL DISTRIBUTION

Binomial distribution was discovered by James Bernoulli (1654-1705) in the year 1700 and was first published posthumously in 1713, eight years after his death. Let a random experiment be performed repeatedly, each repitition being called a trial and let the occurrence of an event in a trial be called a success and its non-occurrence a failure. Consider a set of n independent Bernoullian trials (n being finite) in which the probability 'p' of success in any trial is constant for each trial, then $q = 1 - p$, is the probability of failure in any trial.

The probability of x successes and consequently $(n - x)$ failures in n independent trials, in a specified order (say) $SSFSFFFS...FSF$ (where S represents success and F represents failure) is given by the compound probability theorem by the expression :

$$P(SSFSFFFS...FSF) = P(S)P(S)\,P(F)\,P(S)\,P(F)\,P(F)\,P(F)\,P(S) \times \ldots \times P(F)\,P(S)\,P(F)$$

$$= p \cdot p \cdot q \cdot p \cdot q \cdot q \cdot q \cdot p \ldots q \cdot p \cdot q$$

$$= \underbrace{p \cdot p \cdot p \ldots p}_{\{x \text{ factors}\}} \cdot \underbrace{q \cdot q \cdot q \ldots\ldots q}_{\{(n-x) \text{ factors}\}} = p^x q^{n-x}$$

But x successes in n trials can occur in $\binom{n}{x}$ ways and the probability for each of these ways is same, *viz.*, $p^x q^{n-x}$. Hence the probability of x successes in n trials in *any order* is given by the *addition theorem* of probability by the expression $\binom{n}{x} p^x q^{n-x}$.

The probability distribution of the number of successes, so obtained is called the *Binomial probability distribution*, for the obvious reason that the probabilities of $0, 1, 2, \ldots, n$ successes, *viz.*, $q^n, \binom{n}{1} q^{n-1} p, \binom{n}{2} q^{n-2} p^2, \ldots, p^n$, are the successive terms of the binomial expansion $(q + p)^n$.

Definition. *A random variable X is said to follow binomial distribution if it assumes only non-negative values and its probability mass function is given by :*

$$P(X = x) = p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x} \,;\, x = 0, 1, 2, \ldots, n \,;\, q = 1 - p \\ 0 \qquad\qquad , \textit{otherwise} \end{cases} \qquad \ldots (8{\cdot}3)$$

The two independent constants n and p in the distribution are known as the *parameters* of the distribution. 'n' is also sometimes, known as the degree of the binomial distribution.

Binomial distribution is a discrete distribution as X can take only the integral values, *viz.*, $0, 1, 2, \ldots, n$. Any random variable which follows binomial distribution is known as *binomial variate.*

* Notes:-

$$\binom{n}{x} = {}^nC_x = \frac{n!}{x!(n-x)!}$$

Que] Ten coins are thrown simultaneously. Find the probability of getting at least seven heads. use discrete binomial distribution.

Answer:-

We know that, by discrete binomial distribution -

$$p(x) = \binom{n}{x} p^x q^{n-x}$$

Here we toss 10 coins simultaneously.

$\therefore \quad n=10$

Let p = probability of getting a head $= \frac{1}{2}$

q = probability of not getting a head $= \frac{1}{2}$

$\therefore$ The probability of getting x heads in a ~~random~~ random throw of 10 coins is

$$p(x) = \binom{10}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x}$$

$$\therefore \quad p(x) = \binom{10}{x}\left(\frac{1}{2}\right)^{10} \text{ ----- } x = 0,1,2,\text{-----},9,10$$

$\therefore$ probability of getting at least seven heads is given by

$$P(X \geq 7) = p(7) + p(8) + p(9) + p(10)$$

$$\therefore P(X \geq 7) = \binom{10}{7}\left(\frac{1}{2}\right)^{10} + \binom{10}{8}\left(\frac{1}{2}\right)^{10} + \binom{10}{9}\left(\frac{1}{2}\right)^{10} + \binom{10}{10}\left(\frac{1}{2}\right)^{10}$$

$$\therefore P(X \geq 7) = \left(\frac{1}{2}\right)^{10}\left[\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right]$$

$$\therefore P(X \geq 7) = \left(\frac{1}{2}\right)^{10}\left[\frac{10!}{7! \times (10-7)!} + \frac{10!}{8! \times (10-8)!} + \frac{10!}{9! \times (10-9)!} + \frac{10!}{10! \times (10-10)!}\right]$$

$$\therefore P(X \geq 7) = (0.5)^{10}\left[\frac{10!}{7! \times 3!} + \frac{10!}{8! \times 2!} + \frac{10!}{9! \times 1!} + \frac{10!}{10! \times 0!}\right]$$

$$\therefore P(X \geq 7) = (0.5)^{10}\left[\frac{10 \times 9 \times 8 \times 7!}{7! \times 6} + \frac{10 \times 9 \times 8!}{8! \times 2} + \frac{10 \times 9!}{9! \times 1} + \frac{1}{1}\right]$$

$$\therefore P(X \geq 7) = (0.5)^{10}\left[\frac{10 \times 9 \times 8}{6} + \frac{10 \times 9}{2} + \frac{10}{1} + \frac{1}{1}\right]$$

$$\therefore P(X \geq 7) = (0.5)^{10}\left[\frac{720}{6} + \frac{90}{2} + 10 + 1\right]$$

$$= (0.5)^{10}\left[120 + 45 + 10 + 1\right]$$

$$= (0.5)^{10} \times 176 = 0.0009 \times 176$$

$$\therefore \boxed{P(X \geq 7) = 0.1584}$$

Que] A and B play a game in which their chances of winning are in the ratio 3:2. Find A's chance of winning at least three games out of five games played.

Answer:-

Let p be the probability that "A" wins the game.

Here, Total no. of games = 5

$\therefore n = 5$

$\therefore p = \frac{3}{5} \Rightarrow q = 1 - p = 1 - \frac{3}{5} = 2/5$

Hence, by binomial probability law, the probability that out of 5 games played, A wins x games is given by -

$$P(x) = \binom{5}{x} p^{x} q^{n-x} = \binom{5}{x}\left(\frac{3}{5}\right)^{x}\left(\frac{2}{5}\right)^{5-x}$$

where $x = 0,1,2,3,4,5$

$\therefore$ The required probability that A wins at least three games is given by

$$P(X \geq 3) = P(3) + P(4) + P(5)$$

$$\therefore P(X \geq 3) = \binom{5}{3}\left(\frac{3}{5}\right)^{3}\left(\frac{2}{5}\right)^{5-3} + \binom{5}{4}\left(\frac{3}{5}\right)^{4}\left(\frac{2}{5}\right)^{5-4} + \binom{5}{5}\left(\frac{3}{5}\right)^{5}\left(\frac{2}{5}\right)^{5-5}$$

$$\therefore P(X \geq 3) = \left[{}^5C_3 \times 0.6^3 + 0.4^2\right] + \left[{}^5C_4 \times 0.6^4 \times 0.4^1\right] + \left[{}^5C_5 \times 0.6^5 \times 0.4^0\right]$$

$$\therefore P(X \geq 3) = \left[\frac{5!}{3! \times (5-3)!} \times 0.216 \times 0.16\right] + \left[\frac{5!}{4! \times (5-4)!} \times 0.1296 \times 0.4\right] + \left[\frac{5!}{5! \times (5-5)!} \times 0.0777 \times 1\right]$$

$$\therefore P(X \geq 3) = \left[\frac{5!}{3! \times 2!} \times 0.0346\right] + \left[\frac{5!}{4! \times 1!} \times 0.0518\right] + \left[\frac{5!}{5! \times 0!} \times 0.0777\right]$$

$$\therefore P(X \geq 3) = \left[\frac{5 \times 4 \times 3!}{3! \times 2} \times 0.0346\right] + \left[\frac{5 \times 4!}{4! \times 1} \times 0.0518\right] + \left[\frac{1}{1} \times 0.0777\right]$$

$$\therefore P(X \geq 3) = \left[\frac{20}{2} \times 0.0346\right] + \left[\frac{5}{1} \times 0.0518\right] + \left[0.0777\right]$$

$$\therefore P(X \geq 3) = 0.346 + 0.259 + 0.0777$$

$$\therefore \boxed{P(X \geq 3) = 0.6827}$$

*Que] In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution.

Solution:- It is given that,

$n=5$, $p(1) = 0.4096$ and $p(2) = 0.2048$

According to the Binomial probability law

$$P(X=x) = p(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0,1,2,----5$$

But $q = 1-p$

$$\therefore P(X=x) = p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\therefore p(1) = \binom{5}{1} p^1 (1-p)^{5-1}$$

$$\therefore 0.4096 = \binom{5}{1} p (1-p)^4$$

$$\therefore 0.4096 = 5C_1 \times p \times (1-p)^4$$

$$\therefore 0.4096 = \frac{5!}{1! \times (5-1)!} \times p \times (1-p)^4$$

$$\therefore 0.4096 = \frac{5!}{1! \times 4!} \times p \times (1-p)^4$$

$$\therefore 0.4096 = \frac{5 \times 4!}{1! \times 4!} \times p \times (1-p)^4$$

$$\therefore 0.4096 = 5 \times p \times (1-p)^4 \longrightarrow (1)$$

and $P(2) = \binom{5}{2} p^2 (1-p)^{5-2} = \binom{5}{2} p^2 (1-p)^3$

$\therefore 0.2048 = {}^5C_2 \times p^2 \times (1-p)^3$

$\therefore 0.2048 = \frac{5!}{2! \times (5-2)!} \times p^2 \times (1-p)^3$

$\therefore 0.2048 = \frac{5!}{2! \times 3!} \times p^2 \times (1-p)^3$

$\therefore 0.2048 = \frac{5 \times 4 \times 3!}{2 \times 3!} \times p^2 \times (1-p)^3$

$\therefore 0.2048 = 10 \times p^2 \times (1-p)^3 \rightarrow (2)$

Now, divide equ^n (1) by equ^n (2)

$\therefore \frac{0.4096}{0.2048} = \frac{5 \times p \times (1-p)^4}{10 \times p^2 \times (1-p)^3}$

$\therefore 2 = \frac{1-p}{2p}$

$\therefore 4p = 1-p \Rightarrow 4p + p = 1 \Rightarrow 5p = 1$

$\therefore \boxed{p = \frac{1}{5} = 0.2}$

Que] With the usual notations, Find p for a discrete binomial variate X, if $n=6$ and $9P(X=4) = P(X=2)$

Solution:- We know that, for a discrete binomial distribution

$$P(X=x) = \binom{n}{x} p^x q^{n-x}$$

Here $n=6$

$$\therefore \quad P(X=x) = \binom{6}{x} p^x q^{6-x}, \text{ for } x = 0,1,2,\ldots,6$$

It is given that,

$$9P(X=4) = P(X=2)$$

$$\therefore \quad 9\binom{6}{4} p^4 q^{6-4} = \binom{6}{2} p^2 q^{6-2}$$

$$\therefore \quad 9\binom{6}{4} p^4 q^2 = \binom{6}{2} p^2 q^4$$

$$\therefore \quad 9 \times 6C_4 \times p^4 \times q^2 = 6C_2 \times p^2 \times q^4$$

$$\therefore \quad 9 \times \frac{6!}{4! \times (6-4)!} \times p^4 \times q^2 = \frac{6!}{2! \times (6-2)!} \times p^2 \times q^4$$

$$\therefore 9 \times \frac{6!}{4! \times 2!} \times p^4 \times q^2 = \frac{6!}{2! \times 4!} \times p^2 \times q^4$$

$$\therefore 9p^4q^2 = p^2q^4$$

$$\therefore 9 = \frac{p^2q^4}{p^4q^2} = \frac{q^2}{p^2}$$

$$\therefore 9 = \frac{q^2}{p^2} \Rightarrow 9p^2 = q^2$$

But $q = 1 - p$

$$\therefore 9p^2 = (1-p)^2$$

$$\therefore 9p^2 = 1 - 2p + p^2$$

$$\therefore 9p^2 - p^2 + 2p - 1 = 0$$

$$\therefore 8p^2 + 2p - 1 = 0$$

Which is of the form $ap^2 + bp + c = 0$

Here $a = 8,\ b = 2,\ c = -1$

$$\therefore p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore p = \frac{-2 \pm \sqrt{(2)^2 - 4(8)(-1)}}{2(8)}$$

$$\therefore p = \frac{-2 \pm \sqrt{4 + 32}}{16} = \frac{-2 \pm \sqrt{36}}{16}$$

$$\therefore p = \frac{-2 \pm 6}{16}$$

$$\therefore p = \frac{-2+6}{16} \quad \text{or} \quad p = \frac{-2-6}{16}$$

$$\therefore p = \frac{4}{16} \quad \text{or} \quad p = \frac{-8}{16}$$

$$\therefore p = \frac{1}{4} \quad \text{or} \quad p = -\frac{1}{2}$$

since probability can not be negative.

$$\therefore p \neq -\frac{1}{2}$$

$$\therefore \boxed{p = \frac{1}{4}}$$

* <u>mean and variance for discrete Binomial Distribution:-</u>

IF $X \sim B(n, p)$ then

$$\boxed{\text{mean} = np}$$

$$\boxed{\text{variance} = npq}$$

* <u>Note:-</u> $\binom{n}{0} = {}^nC_0 = 1$

Que] The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(X \geq 1)$.

Solution -

Let $X \sim B(n,p)$

We are given that,

$$\text{mean} = np = 4 \rightarrow (1)$$

$$\text{variance} = npq = 4/3 \rightarrow (2)$$

Divide equation (2) by (1)

$$\therefore \frac{npq}{np} = \frac{4/3}{4} = \frac{4}{4 \times 3}$$

$$\therefore \boxed{q = 1/3}$$

$$\therefore p = 1 - q = 1 - 1/3 = 2/3$$

$$\therefore \boxed{p = 2/3}$$

For discrete binomial distribution we know that

$$P(X = x) = \binom{n}{x} p^{x} q^{n-x}$$

$$\therefore P(X \geq 1) = 1 - P(X = 0)$$

$$P(X \geq 1) = 1 - \left[\binom{n}{0} p^{0} q^{n-0}\right]$$

$\therefore P(X \geq 1) = 1 - [1 \times 1 \times q^n]$

$\therefore P(X \geq 1) = 1 - q^n \longrightarrow (3)$

consider, equation (2)

(2) $\Rightarrow \quad npq = 4/3$

$\therefore n \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{3}$

$\therefore n \times \frac{2}{9} = \frac{4}{3} \Rightarrow n = \frac{4}{3} \times \frac{9}{2} = 6$

$\therefore \boxed{n = 6}$

$\therefore$ (3) $\Rightarrow \quad P(X \geq 1) = 1 - q^n = 1 - \left(\frac{1}{3}\right)^6$

$\therefore P(X \geq 1) = 1 - (0.3333)^6$
$= 1 - 0.0014$
$= 0.9986$

$\therefore \boxed{P(X \geq 1) = 0.9986}$

* Poission Distribution :-

Poission distribution was discovered by the French mathematician and physicist Simeon Dennis Poission who published it in 1837.

Poission distribution is a limiting case of the binomial distribution under the following conditions.

① n, the number of trials is indefinitely large i.e. $n \to \infty$

② p, the constant probability of success for each trial is indefinitely small i.e. $p \to 0$

③ $np = \lambda$ is finite (is a mean of distribution)

Thus

$$p = \frac{\lambda}{n} \quad \text{and} \quad q = 1 - \frac{\lambda}{n}$$

where λ is a positive real number.

Definition :- A random variable X is said to be follow a poisson distribution if it assumes only non-negative values and it's PMF is given by -

$$P(x, \lambda) = P(X = x) = \begin{cases} \dfrac{e^{-\lambda} \lambda^x}{x!} ; & x = 0, 1, 2, \ldots \\ 0 ; & \text{otherwise} \end{cases}$$

Here $\lambda > 0$

Que] A manufacturer of cotter pins knows that 5% of his product is defective. If he sells cotter pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality? Use poission distribution.

Solution:-

It is given that, 5% product is a defective.

$\therefore$ p = probability of a defective pins $= 5\% = 0.05$

It is also given that, manufacturer sells cotter pins in boxes of 100.

$\therefore$ $n = 100$

$\therefore$ λ = mean number of defective pins

$= np = 100 \times 0.05 = 5$

$\therefore$ $\lambda = 5$

Let, the random variable x denote the number of defective pins in a box of 100.

Then, by poission probability law; the probability of x defective pins in a box is

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-5}5^{x}}{x!} \text{ where } x=0,1,2,\ldots$$

$\therefore$ probability that a box will fail to meet the guaranteed quality is –

$$P(X>10) = 1 - P(X \leq 10)$$

$$= 1 - \sum_{x=0}^{10} \frac{e^{-5}5^{x}}{x!}$$

$$\therefore P(X>10) = 1 - e^{-5}\sum_{x=0}^{10}\frac{5^{x}}{x!}$$

$$\therefore P(X>10) = 1 - e^{-5}\left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!} + \frac{5^5}{5!} + \frac{5^6}{6!} + \frac{5^7}{7!} + \frac{5^8}{8!} + \frac{5^9}{9!} + \frac{5^{10}}{10!}\right]$$

$$\therefore P(X>10) = 1 - e^{-5}\left[\frac{1}{1} + \frac{5}{1} + \frac{25}{2} + \frac{125}{6} + \frac{625}{24} + \frac{3125}{120} + \frac{15,625}{720} + \frac{78,125}{5040} + \frac{3,90,625}{40,320} + \frac{1,953,125}{3,62,880} + \frac{97,65,625}{36,28,800}\right]$$

$$\therefore P(X>10) = 1 - e^{-5}\left[1 + 5 + 12.5 + 20.83 + 26.04 + 26.04 + 21.70 + 15.50 + 9.69 + 5.38 + 2.69\right]$$

$$\therefore P(X>10) = 1 - e^{-5}(146.37)$$

$$= 1 - (0.0067 \times 146.37)$$

$$= 1 - 0.9807$$

$$= 0.0193$$

$$\therefore \boxed{P(X>10) = 0.0193}$$

QUE] A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. calculate the proportion of days on which

① Neither car is used

② The proportion of days on which some demand is refused. (use more than 2 cars)

Solution:- Here the random variable X, which denotes the number of demands for a car on any day follows poisson distribution with mean $\lambda = 1.5$.

The proportion of days on which there are x demands for a car is given by

$$P(X=x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \; ; \; x = 0, 1, 2, \text{-----}$$

$$\therefore P(X=x) = \frac{e^{-1.5} (1.5)^{x}}{x!}$$

① proportion of days on which neither car is used is given by

$$P(X=0) = \frac{e^{-1.5}(1.5)^0}{0!} = \frac{e^{-1.5} \times 1}{1} = e^{-1.5}$$

$$\therefore \boxed{P(X=0) = 0.2231}$$

② proportion of days on which some demand is refused is (use more than 2 cars)

$$P(X>2) = 1 - P(X \le 2)$$

$$= 1 - \sum_{x=0}^{2} \frac{e^{-1.5}(1.5)^x}{x!}$$

$$\therefore P(X>2) = 1 - e^{-1.5} \sum_{x=0}^{2} \frac{(1.5)^x}{x!}$$

$$= 1 - 0.2231\left[\frac{(1.5)^0}{0!} + \frac{(1.5)^1}{1!} + \frac{(1.5)^2}{2!}\right]$$

$$= 1 - 0.2231\left[\frac{1}{1} + \frac{1.5}{1} + \frac{2.25}{2}\right]$$

$$= 1 - 0.2231\left[1 + 1.5 + 1.125\right]$$

$$= 1 - (0.2231 \times 3.625)$$

$$= 1 - 0.8087 = 0.1913$$

$$\therefore \boxed{P(X>2) = 0.1913}$$

Que] An insurance company insures 4,000 people against loss of both eyes in a car accident. Based on previous data, the rates were computed on the assumptions that on the average of 10 persons in 1,00,000 will have car accident each year that result in this type of injury. What is the probability that more than 3 of the insured will collect on their policy in a given year? use poisson distribution.

Solution:- It given that,

An insurance company insures 4,000 people against loss of both eyes in a car accident.

$\therefore \boxed{n = 4,000}$

p = probability of loss of both eyes in a car accident $= \frac{10}{1,00,000} = 0.0001$

$\therefore \boxed{p = 0.0001}$

The parameter λ of the poisson distribution is

$$\lambda = np = 4000 \times 0.0001 = 0.4$$

$\therefore \boxed{\lambda = 0.4}$

Let the random variable X denote number of car accidents in the batch of 4,000 people. Then by poisson probability law

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.4}(0.4)^{x}}{x!}, \quad x = 0,1,2,-----$$

Hence the required probability that, more than 3 of the insured will collect on their policy is given by –

$$P(X>3) = 1 - P(X \leq 3)$$

$$= 1 - \sum_{x=0}^{3} \frac{e^{-0.4}(0.4)^{x}}{x!}$$

$$= 1 - e^{-0.4} \sum_{x=0}^{3} \frac{(0.4)^{x}}{x!}$$

$$\therefore P(X>3) = 1 - e^{-0.4}\left[\frac{(0.4)^{0}}{0!} + \frac{(0.4)^{1}}{1!} + \frac{(0.4)^{2}}{2!} + \frac{(0.4)^{3}}{3!}\right]$$

$$= 1 - e^{-0.4}\left[\frac{1}{1} + \frac{0.4}{1} + \frac{0.16}{2} + \frac{0.064}{6}\right]$$

$$= 1 - e^{-0.4}\left[1 + 0.4 + 0.08 + 0.0107\right]$$

$$= 1 - e^{-0.4}\left[1.4907\right]$$

$$= 1 - (0.6703 \times 1.4907)$$

$$= 1 - 0.9992 = 0.0008$$

$$\therefore \boxed{P(X>3) = 0.0008}$$