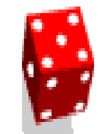
Probabilistic models

Probability

- ✓ **Experiment:** a procedure involving chance that leads to different results.
- ✓ **Outcome:** the result of a single trial of an experiment
- ✓ **Event:** one or more outcomes of an experiment
- ✓ **Probability:** the measure of how likely an event is;

Example: a fair 6-sided dice



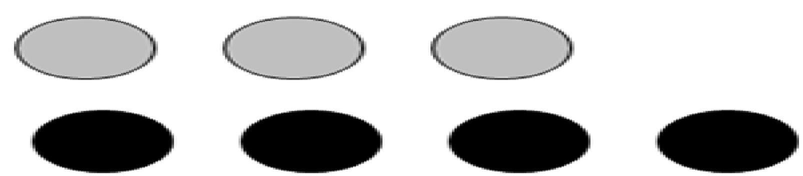
- ✓ Outcome: The possible outcomes of this experiment are 1, 2, 3, 4, 5 and 6;
- ✓ Events: 1; 6; even
- ✓ Probability: outcomes are ***equally likely*** to occur.
  - ✓ P(A) = The Number Of Ways Event A Can Occur / The Total Number Of Possible Outcomes
  - ✓ P(1)=P(6)=1/6; P(even)=3/6=1/2;

Conditional Probability

- $P(i|\theta)$: the measure of how likely an event i happens under the condition θ;
  - Example: two dices D1, D2
    - $P(i|D1)$ →probability for picking i using dicer D1
    - $P(i|D2)$ →probability for picking i using dicer D2

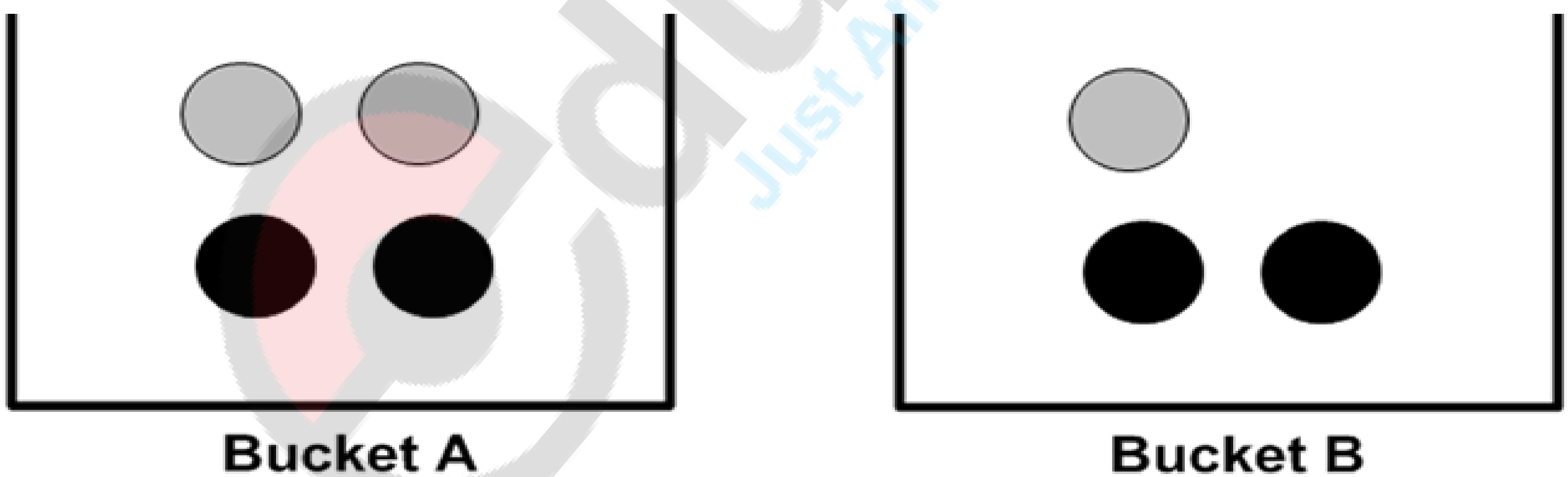
Conditional probability

- Let’s assume for a moment that we have a jar containing seven stones.
- Three of these stones are gray and four are black, as shown in figure.
- If we stick a hand into this jar and randomly pull out a stone, what are the chances that the stone will be gray?
- There are seven possible stones and three are gray, so the probability is 3/7.
- Black 4/7.



Conditional probability

- We calculated the probability of drawing a gray stone P(gray) by counting the number of gray stones and dividing this by the total number of stones.
- What if the seven stones were in two buckets?
- This is shown in figure



Conditional probability

- If you want to calculate the P(gray) or P(black), would knowing the bucket change the answer?
- If you wanted to calculate the probability of drawing a gray stone from bucket B, you could probably figure out how do to that. This is known as ***conditional probability.***

- We're calculating the probability of a gray stone, given that the unknown stone comes from bucket B.

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Conditional probability

- It’s not hard to see that P(gray|bucketA) is 2/4 and P(gray|bucketB) is 1/3.
- To formalize how to calculate the conditional probability, we can say P(gray|bucket B) = P(gray and bucket B)/P(bucket B)
- P(gray and bucketB) = 1/7.
- P(bucketB) is 3/7.
- P(gray|bucketB) = P(gray and bucketB)/P(bucketB) = (1/7) / (3/7) = 1/3.

Conditional probability

- Another useful way to manipulate conditional probabilities is known as Bayes' rule.
- Bayes' rule tells us how to swap the symbols in a conditional probability statement.
- If we have P(x|h) but want to have P(h|x), we can find it with the following:

$$P(H \mid X) = \frac{P(X \mid H)P(H)}{P(X)}$$

Classifying with conditional probabilities

- Bayesian decision theory told us to find the two probabilities:
- If p1(x, y) > p2(x, y), then the class is 1.
- If p2(x, y) > p1(x, y), then the class is 2.
- we really need to compare are p(c1|x,y) and p(c2|x,y).
- Let's read these out to emphasize what they mean.
- Given a point identified as x,y, what is the probability it came from class c1? What is the probability it came from class c2?.

Classifying with conditional probabilities

- we can define the Bayesian classification rule:
- If $P(c1|x, y) > P(c2|x, y)$, the class is c1.
- If $P(c1|x, y) < P(c2|x, y)$, the class is c2.