Functional Dependency Problems

1. For relation R{A,B,C,D,E,F,G}

{ $A \rightarrow B$, $BC \rightarrow DE$, $AEF \rightarrow G$ } $\models ACF \rightarrow DG$

Source of the RHS

So we need to show that $ACF \rightarrow BC$ and AEF

Notice $A \rightarrow B$ can be augmented to $AC \rightarrow BC$ that transitively yields DE.

By the 'shootover rule' (augmentation),
$AC \rightarrow DE$ yields $AC \rightarrow ACDE$

And with more augmentation we get
$ACF \rightarrow ACDEF$

1. For relation R{A,B,C,D,E,F,G}

$\{ A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G \} \models ACF \rightarrow DG$

1. $A \rightarrow B$ (given)
2. $BC \rightarrow DE$ (given)
3. $AEF \rightarrow G$ (given)
4. $AC \rightarrow BC$ (1, aug.)
5. $AC \rightarrow DE$ (4,2 trans)
6. $ACF \rightarrow DEF$ (5, aug.)
7. $ACF \rightarrow ACDEF$ (6, aug.)
8. $ACF \rightarrow AEF$ (7, decomp)
9. $ACF \rightarrow G$ (8,3 trans)
10. $ACF \rightarrow D$ (6, decomp)
11. $ACF \rightarrow DG$ (9,10 union)

2. For R {ABCDEF}

$\{A \rightarrow BC, B \rightarrow E, CD \rightarrow EF \} \models AD \rightarrow F$

1. $A \rightarrow BC$ (given)
2. $B \rightarrow E$ (given)
3. $CD \rightarrow EF$ (given)
4. $AD \rightarrow BCD$ (1, aug)
5. $AD \rightarrow CD$ (4, decomp)
6. $AD \rightarrow EF$ (5,3 trans)
7. $AD \rightarrow F$ (6, decomp)

NOTE: Proof Strategies

- Suppose you have to prove $AB \rightarrow CD$
- Try deducing a functional dependency with CD on the RHS (right hand side)
  - Augmentation, Union or Decomposition can modify RHS of FDs.
  - result: $x \rightarrow CD$

Strategies (cont.)

- Try deducing a functional dependency with AB on the LHS.
  - Augmentation, Reflexivity, Pseudo-transitivity can affect LHS.
  - result: $AB \rightarrow y$
- Now try to deduce $y \rightarrow x$
- Try disproof first: it is mechanical.

Are 2 sets of FDs Equivalent?

- First method:
  - Compute the closure of F
  - Compute the closure of G
  - See if they are equal
- Second method
  - Show every FD in F can be proven from G
  - Show every FD in G can be proven from F

3. Equivalent sets of FDs ?

$F = \{ B \to CD, AD \to E, B \to A\}$

$G = \{ B \to CDE, B \to ABC, AD \to E\}$

- $F \models G$? (AD → E in both.)

1. $B \to CD$ (given)
2. $B \to A$ (given)
3. $AD \to E$* (given)
4. $B \to ACD$ (1,2, union)
5. $B \to AD$ (4, decomp)
6. $B \to E$ (5,3, trans)
7. $B \to ACDE$ (4,6,union)
8. $B \to CDE$* (7,decomp)
9. $B \to AC$ (4, decomp)
10. $B \to ABC$* (9,aug)

*FD in G to be Proven

3. Continued

$F = \{ B \rightarrow CD, AD \rightarrow E, B \rightarrow A\}$

$G = \{ B \rightarrow CDE, B \rightarrow ABC, AD \rightarrow E\}$

- $G \models F$? (prove F from G?)
- Obviously

4. What is the key for R?

- {KEY} → {REST}
  because key must determine all fields
- KEY must include all NOT on RHS of ANY functional dependency
  - Only fields on RHS are determined.
  - An undetermined field must be in the key.
- REST must include all NOT on LHS
  - They don't determine anything
  - so they can not be part of key.
- If field is not in ANY FD, it must be part of the key

4. What is the key for R?

- R=(A,B,C,D,E,F,G,H,I,J)
  FDs: AB $\rightarrow$ C, BD $\rightarrow$EF, AD $\rightarrow$ GH, A $\rightarrow$ I, H $\rightarrow$ AJ
- NOT on RHS: Key must include BD
- NOT on LHS: Rest must include CEFGIJ
- Unknown: A and H
- Is BD a Key? No: it only determines EF which don't determine anything else
- If BD were a key, we would stop here.
- So Key might include A or H
  - But not both, Why not?

FDs: AB $\rightarrow$ C, BD $\rightarrow$EF, AD $\rightarrow$ GH, A $\rightarrow$ I, H $\rightarrow$ AJ

- Try adding A: is ABD a key?
  - ABD $\rightarrow$ EF because BD $\rightarrow$EF (aug)
  - ABD$\rightarrow$ C because AB$\rightarrow$ C (aug)
  - ABD $\rightarrow$ GH because AD $\rightarrow$ GH (aug)
  - ABD $\rightarrow$ I because A $\rightarrow$ I (aug)
  - ABD $\rightarrow$ J because AD $\rightarrow$ GH & H $\rightarrow$ AJ (aug, decomp, trans)

FDs: $AB \rightarrow C$, $BD \rightarrow EF$, $AD \rightarrow GH$, $A \rightarrow I$, $H \rightarrow AJ$

- Try adding H: is BDH a key?
- Since $H \rightarrow AJ$, $H \rightarrow A$.
- So $BDH \rightarrow BDA$, which is a key.
  - So BDH determines all that ABD determines.
- BDH is another key
- 2 overlapping keys: ABD and BDH.

5. Counter example: $\{ XY \rightarrow Z, Z \rightarrow X \} \models Y \rightarrow XZ$?

- Method: Set up a database in which LHS is NOT Violated but RHS IS violated
- Why? Because $\models$ is a form of implication and implication is only false when LHS is True and RHS is false
- When is RHS ($Y \rightarrow XZ$)false?
  - It too is a kind of implication!
  - When 2 tuples agree in Y and disagree in XZ

counter: $\{ XY \to Z, Z \to X \} \models Y \to XZ$

- Requirements for counter:
- two tuples
  - agree in Y, disagree in X and/or Z
  - do not violate LHS.
- Important Note: given $A \to B$
  - two tuples which disagree in A cannot violate $A \to B$.
  - Why? Because implication is false only when A is true and B is false.

counter: $\{ XY \to Z, Z \to X \} \models Y \to XZ$

- X Y Z (attributes)
  a b c (first tuple)
  ? b ? (y must be the same)
- What about X? If X is the same:

X	Y	Z
a	b	c
a	b	?

- Problem: Cannot violate LHS ($XY \to Z$)
  - so Z must be the same
  - but cannot have 2 identical tuples.
  - Therefore, make X different

counter: $\{ XY \to Z, Z \to X \} \models Y \to XZ$

- X Y Z
  a b c
  H b ? (y the same & x different)
- What about Z? Suppose Z is the same.
  - Maybe OK since $Y \to XZ$ still violated.
  - X Y Z
    a b c
    H b c
- But this would violate the LHS. Why?

counter: $\{ XY \to Z, Z \to X \} \models Y \to XZ$

- Solution:
  X Y Z
  a b c
  H b K
- Check:
  - RHS violated?
    - Yes: Same in Y, different in X and/or Z
  - LHS not violated?
    - $XY \to Z$: no two the same in XY
    - $Z \to X$: no two the same in Z