3D transformation: Translation

1. Translation : matrix for translation with homogeneous coordinate is given as ,

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

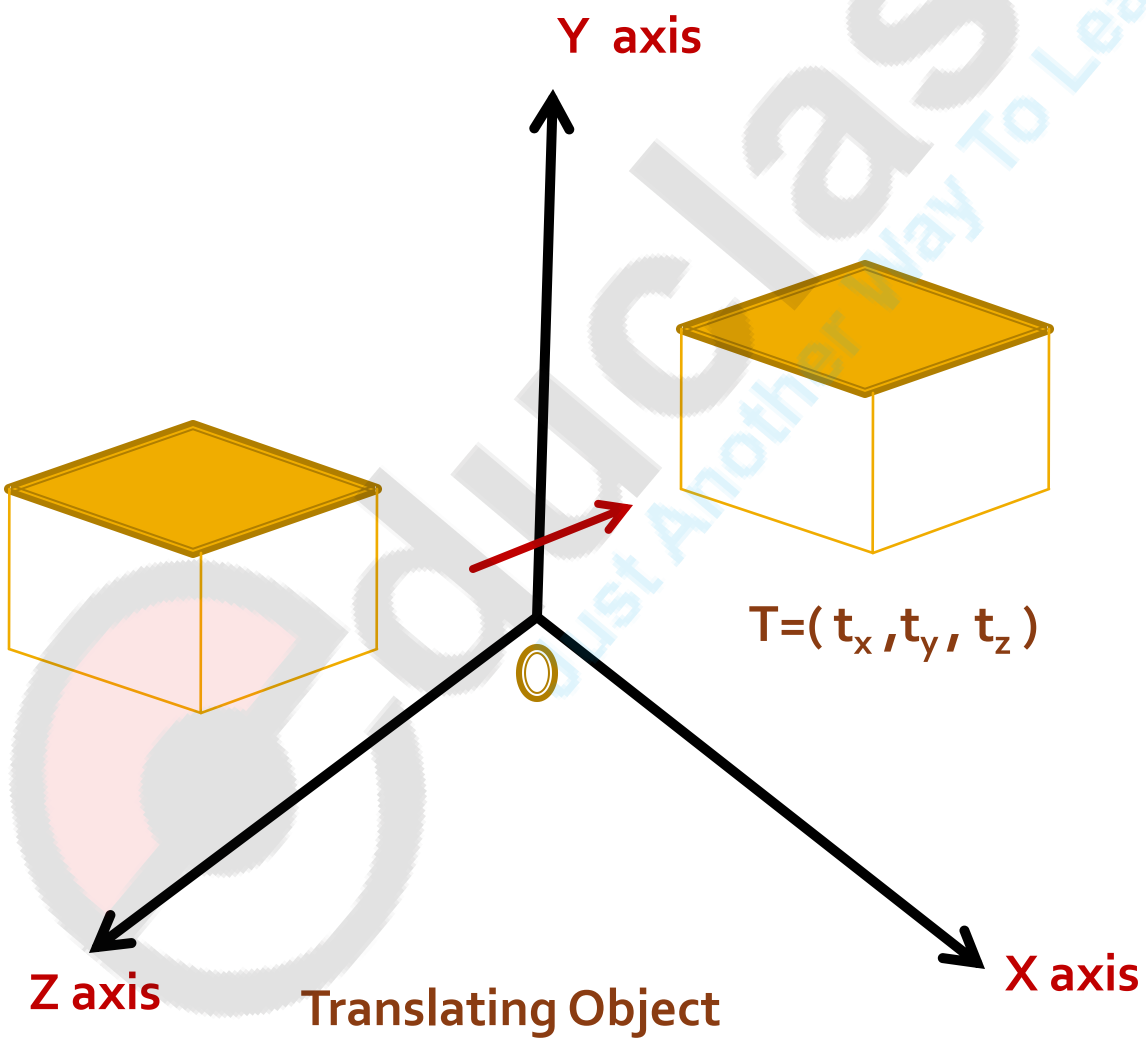
$P' = P \cdot T$

3D transformation : Translation

$$\begin{bmatrix} X' & Y' & Z' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & Z & 1 \end{bmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{pmatrix}$$

$$= \begin{bmatrix} x + t_x & y + t_y & z + t_z & 1 \end{bmatrix}$$

3D transformation : Translation



3D transformation: Scaling

2. **Scaling :** matrix for scaling with homogeneous coordinate is given as ,

$$T=\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

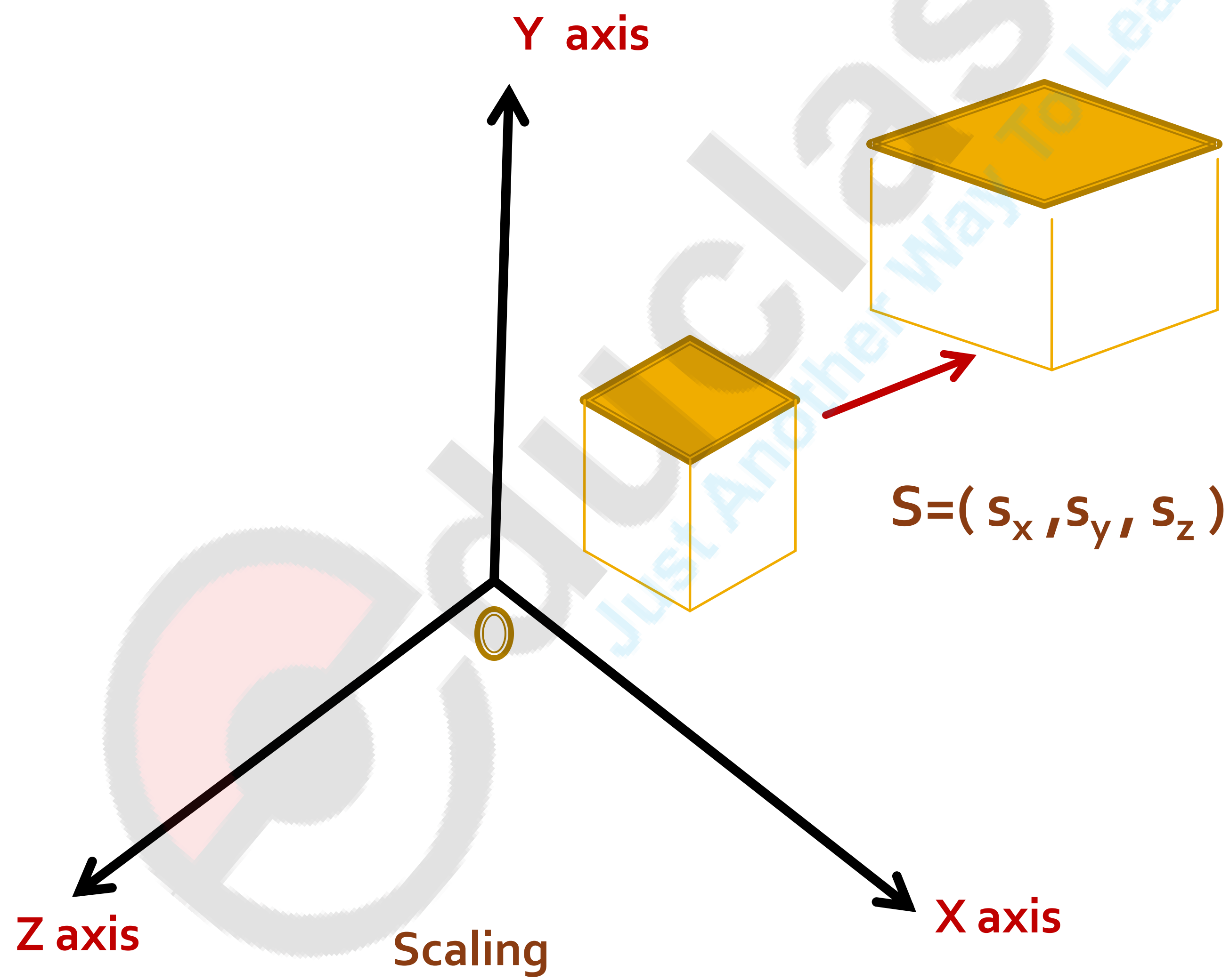
$$P' = P \, . \, S$$

3D transformation: Scaling

$$\begin{bmatrix} X' & Y' & Z' & 1 \end{bmatrix} = \begin{bmatrix} X & Y & Z & 1 \end{bmatrix} \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} x.s_x & y.s_y & z.s_z & 1 \end{bmatrix}$$

3D transformation: Scaling



Scaling of an object with respective to selected point:

- It can be represented with the following transformation sequence:
    1. Translate the fixed point to the origin
    2. Scale the object
    3. Translate the fixed point back to its original position

3D transformation: Rotation about Z axis

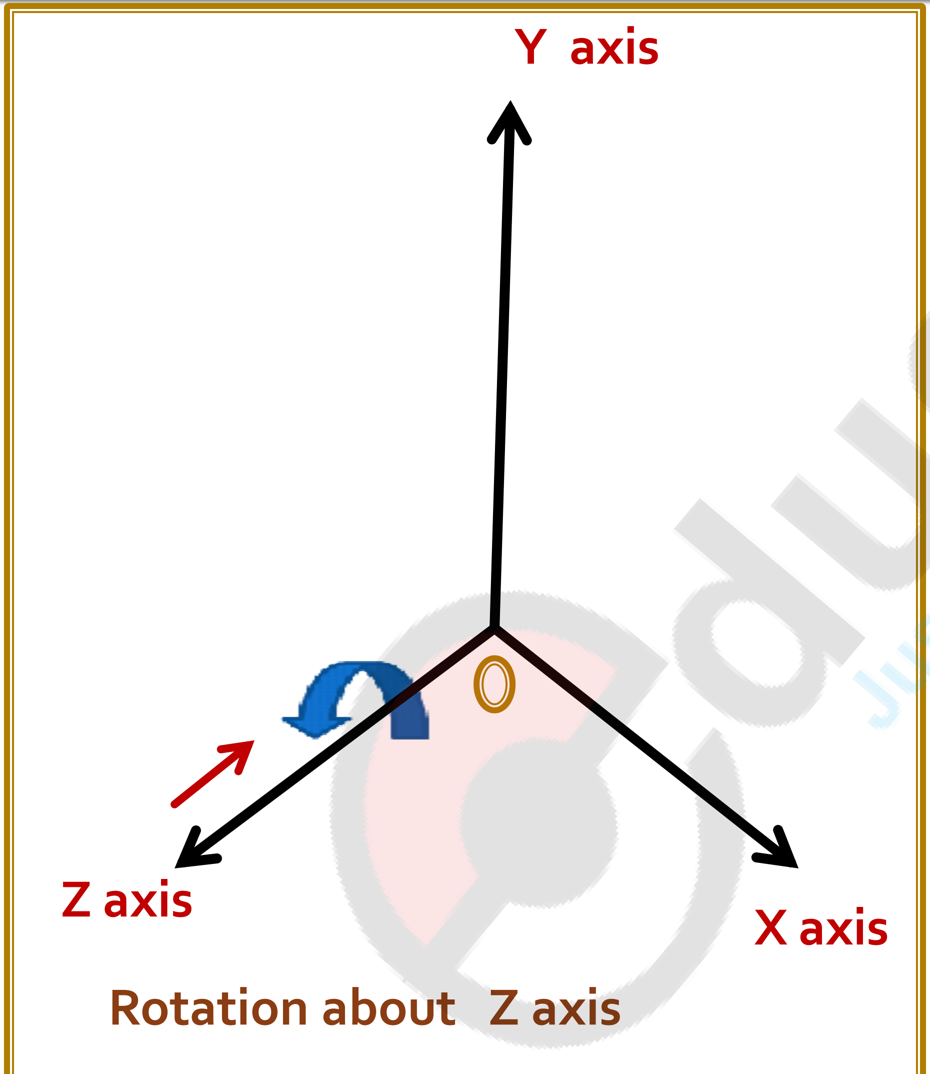
3. **Rotation :** matrix for each coordinate axis rotation with homogeneous coordinate is given as ,

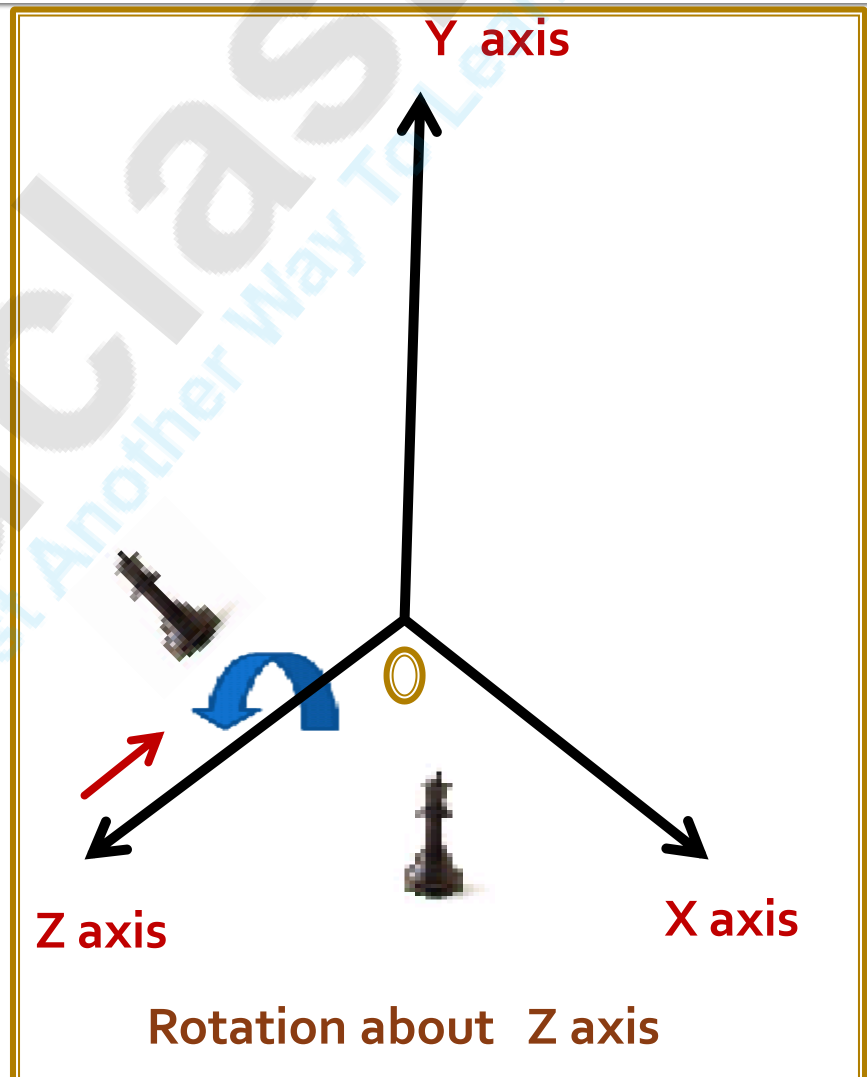
$$R_z = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 & 0 \\ -\sin\Theta & \cos\Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Θ is positive indicate counter clockwise (anticlockwise) Rotation

Otherwise Θ is negative indicate clockwise Rotation

3D transformation: Rotation about Z axis



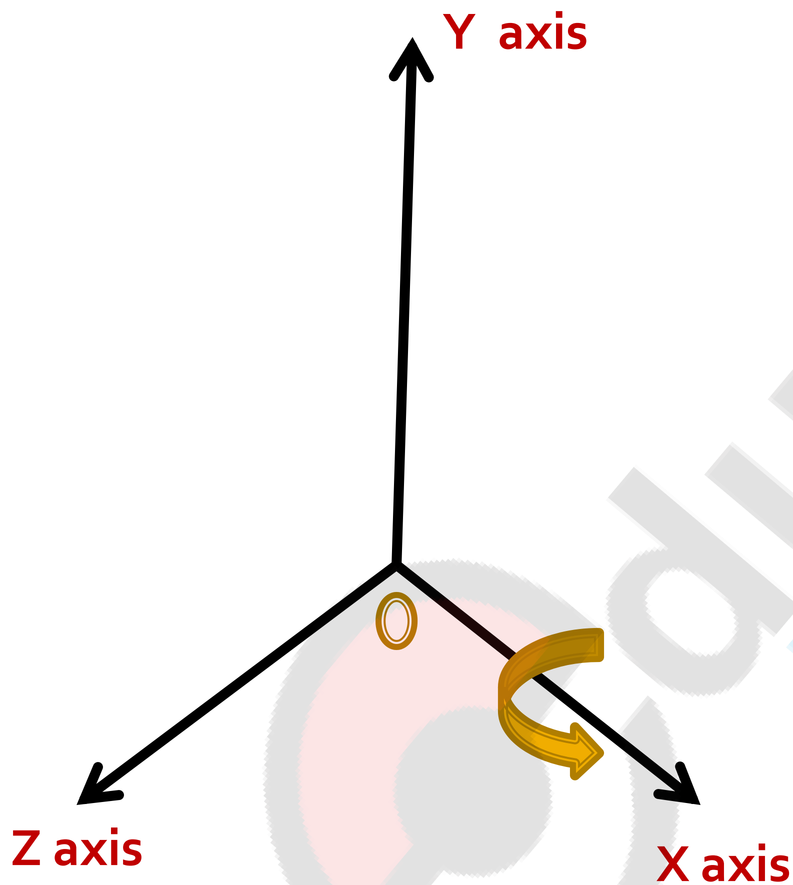


3D transformation: Rotation about x axis

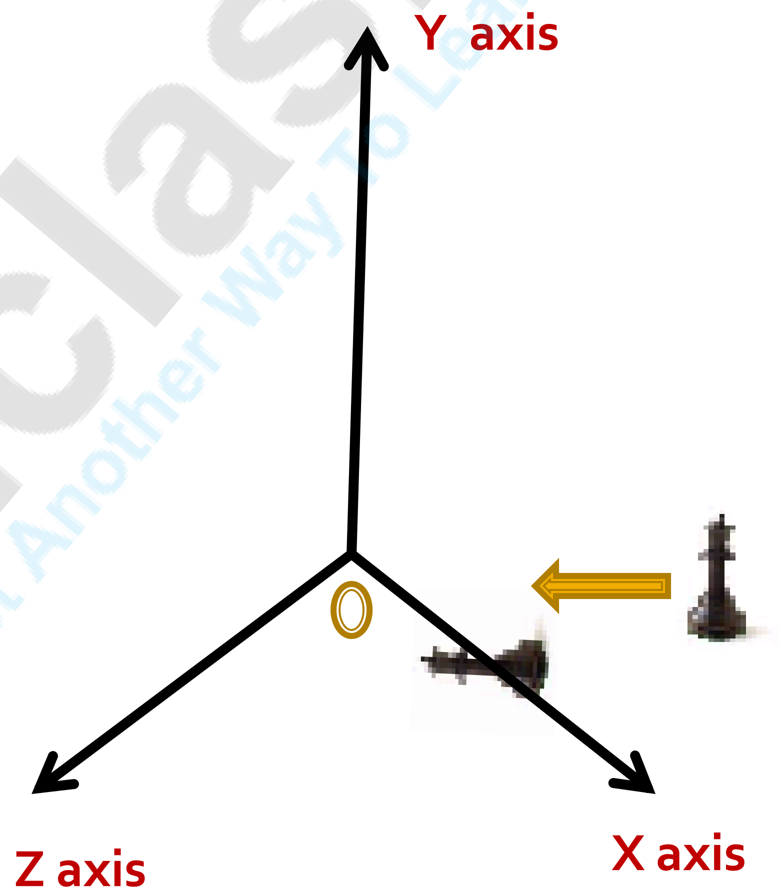
Rotation about x axis

$$Rx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Theta & \sin\Theta & 0 \\ 0 & -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D transformation: Rotation about x axis



Rotation about x axis



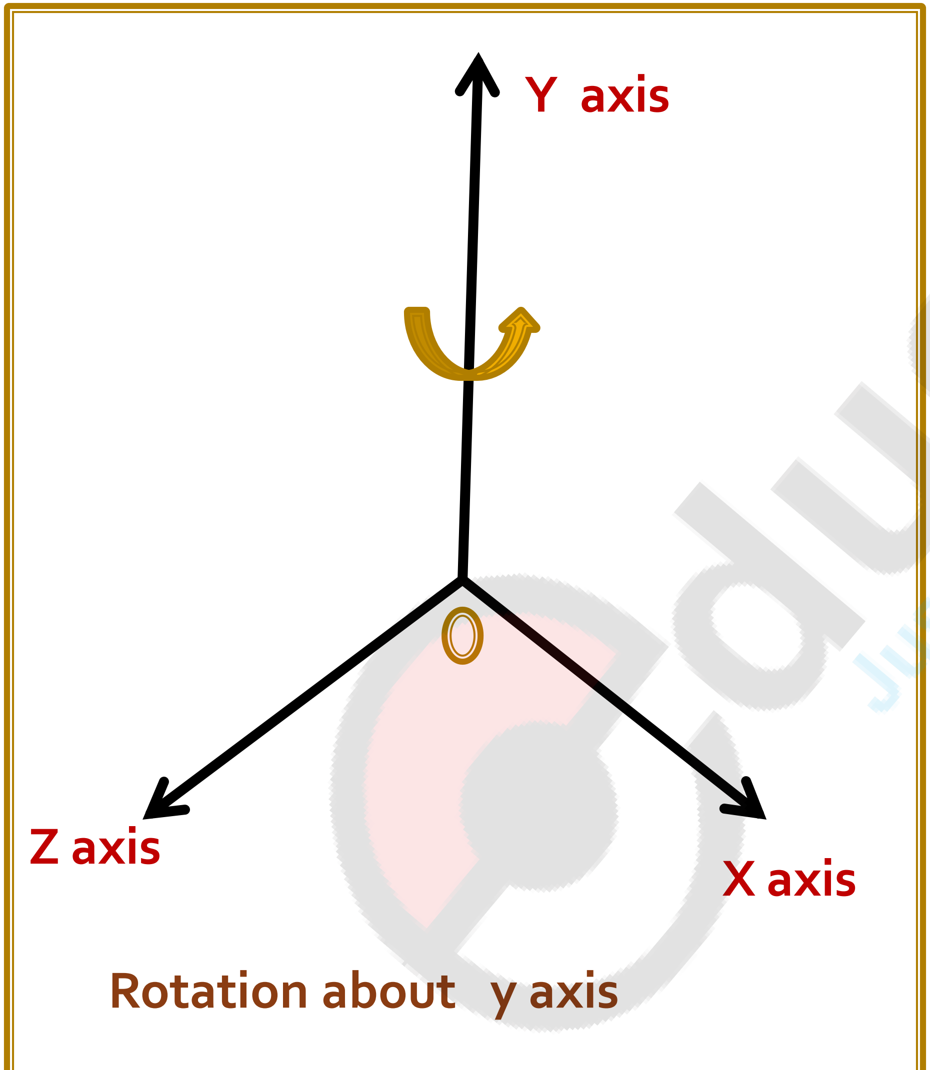
Rotation about x axis

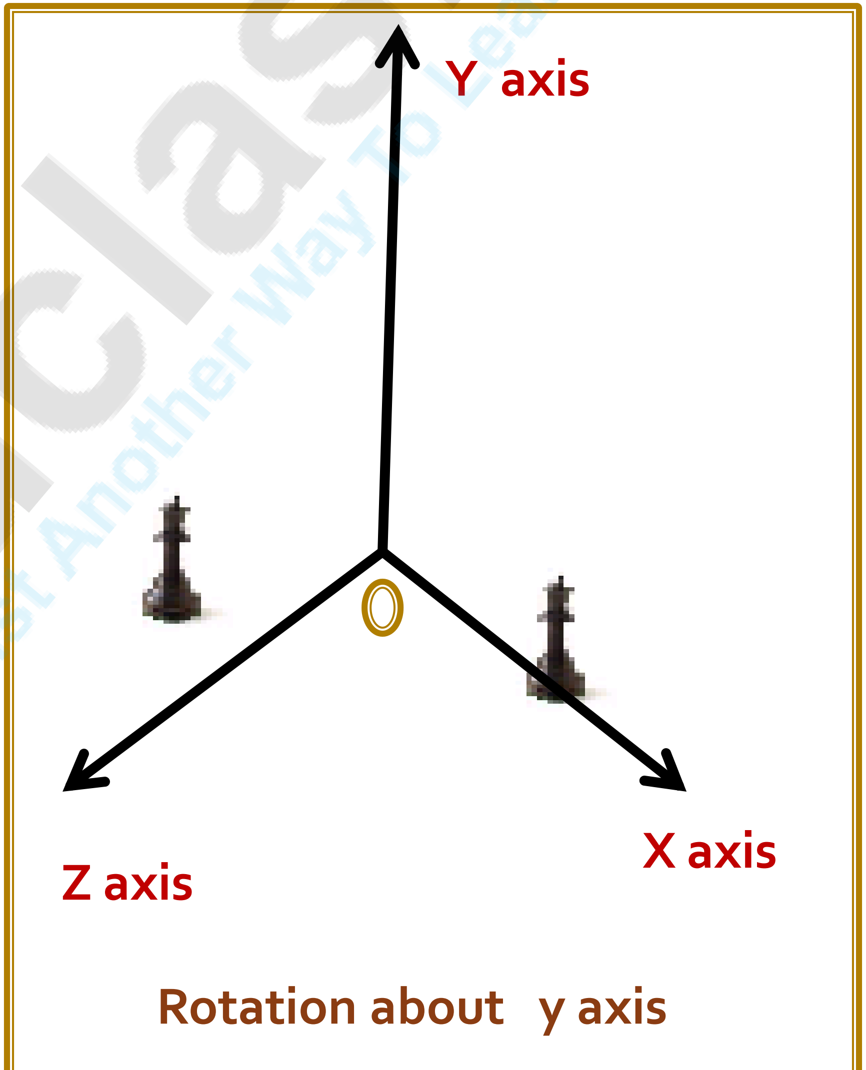
3D transformation: Rotation about Y axis

Rotation about y axis

$$R_y = \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\Theta & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D transformation: Rotation about y axis

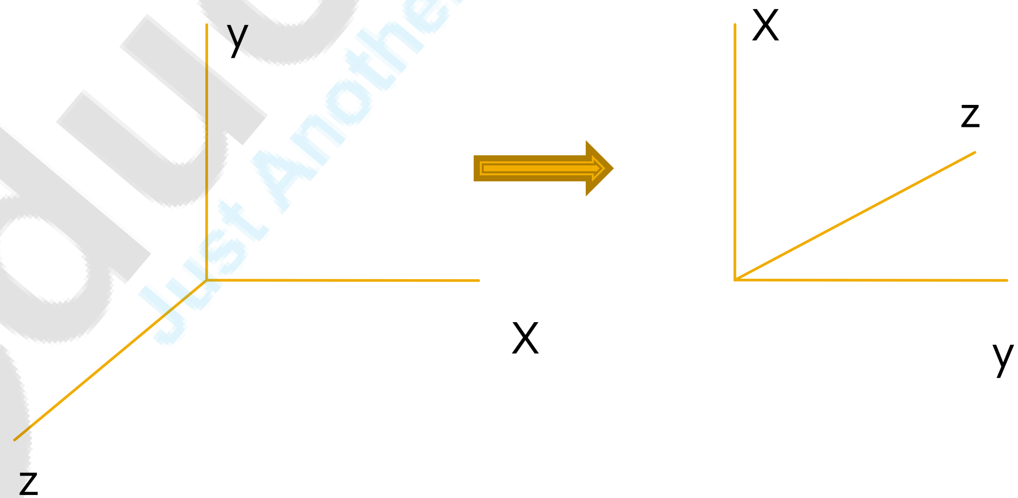




Reflection

- Reflection ralative to XY plane

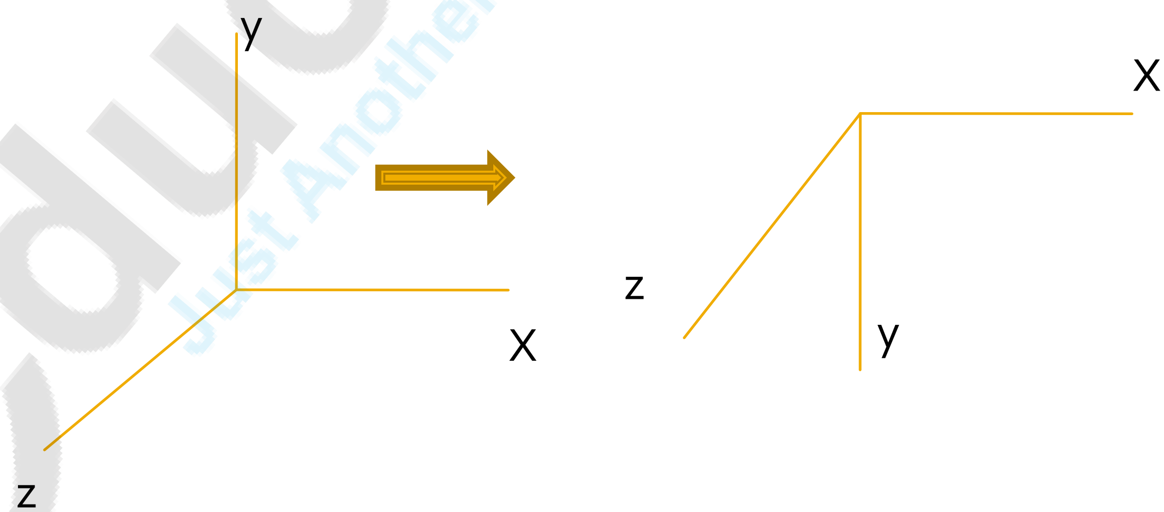
$$RF_{z=}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Reflection

- Reflection ralative to XZ plane

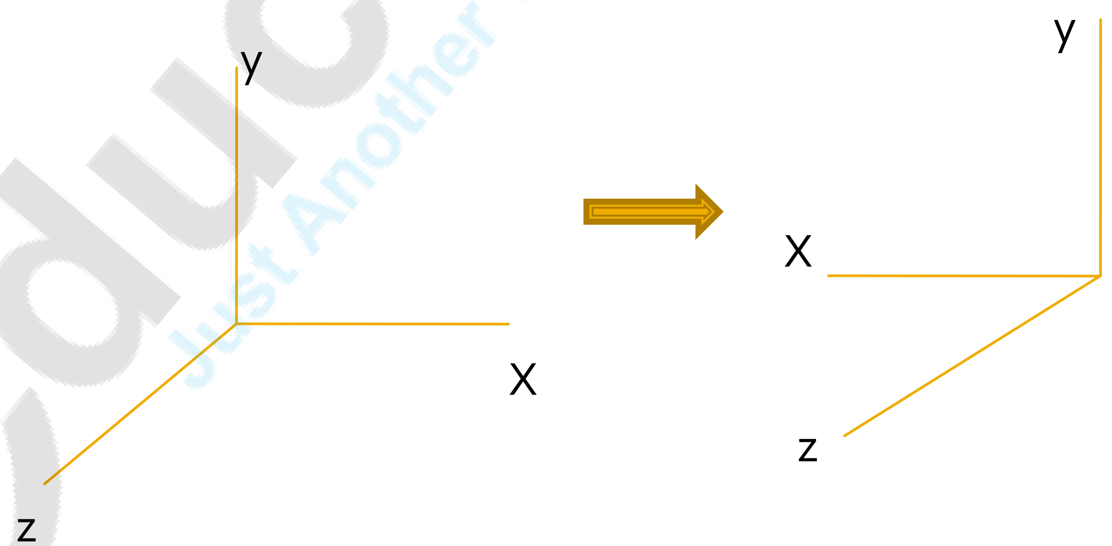
$$RFy = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Reflection

- Reflection ralative to YZ plane

$$RF_{x=} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



shear

- It is used to modify the shapes of objects and they are useful in 3D viewing for obtaining general projection transformation

$$SH_Z=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ a & b & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad SH_X=\begin{bmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_Y=\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problems

Q Explain 3 D object representation ? 10m

Q. Give the 3D transformation matrix for translation ,

scaling , rotation ? 5m

Q. Derive single 4 x 4 matrix for following transformation

Rotation by 180 ° around y axis

Translate by 3 units in x axis and 4 units in y axis

scale by 4 unit in y axis 10m

Rotation by 180 around y axis:

$$R_y = \begin{pmatrix} \cos(180) & 0 & -\sin(180) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(180) & 0 & \cos(180) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\cos(180) = -1$

$\sin(180) = 0$

$$R_y = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Translate by 3 units in x axis and 4 units in y axis

- so , $t_x=3$ $t_y=4$ $t_z=0$

$$T=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{pmatrix}$$

scale by 4 unit in y axis

$$S=\begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$T_{final} = \mathbf{Ry} \ . \mathbf{T} \ . \mathbf{S}$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 3 & 16 & 0 & 1 \end{bmatrix}$$

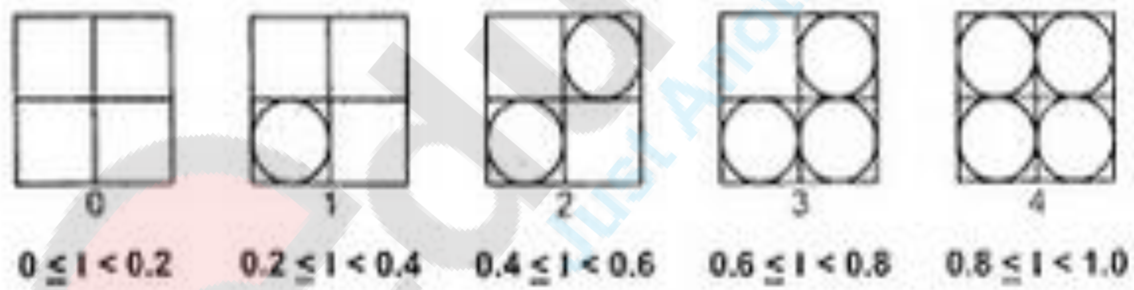
$$[X' \quad Y' \quad Z' \quad 1] = [X \quad Y \quad Z \quad 1] \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 3 & 16 & 0 & 1 \end{bmatrix}$$

$$= [-x + 3 \quad 4y + 16 \quad -z \quad 1]$$

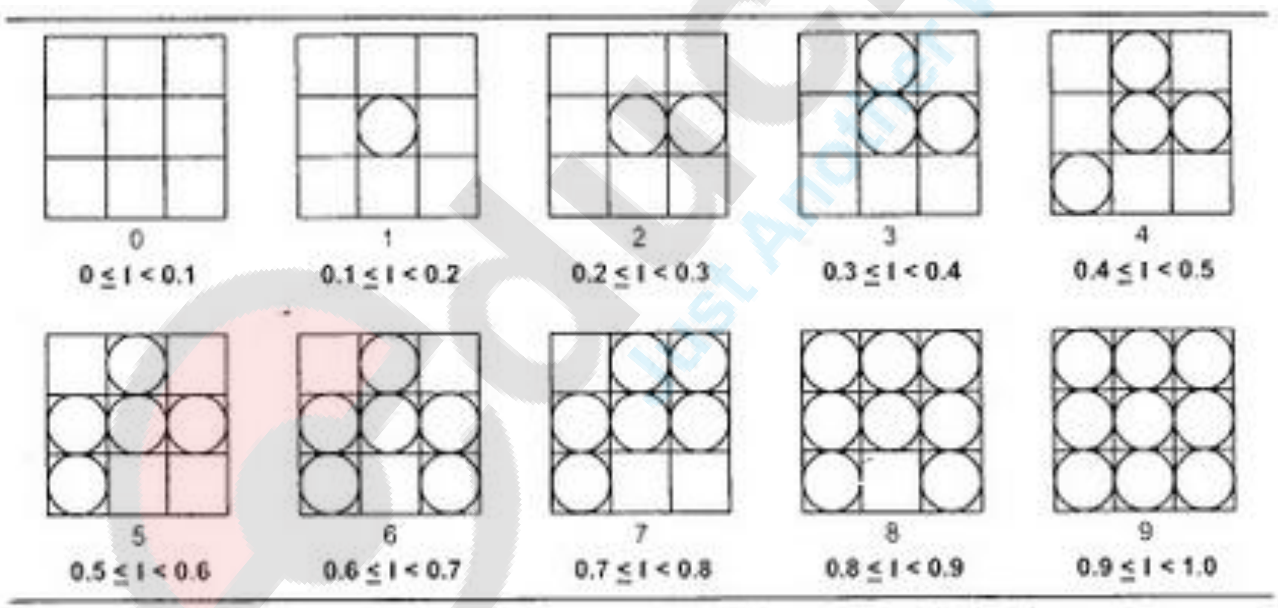
Halftone shading

- Many displays and hardcopy devices are bi-level
- They can only produce two intensity levels
- In such devices we can increase the number of available intensities by considering combine intensity of multiple pixels is known as half toning.
- This is commonly used in printing black and white photographs in newspaper , magazines and books
- The pictures produced by half toning process are called halftones.

- Following figures shows 2 X 2 halftone patterns



- Following figures shows 3 X 3 halftone patterns



Dithering Techniques

- It refers to the technique for approximating halftones without reducing resolution
- It is also applied to halftone approximation methods using pixel grids
- Random values added to pixel intensities to break up contours are often refer as dither noise.
- To obtain n^2 intensity levels ,it is necessary to setup an nXn dither matrix Dn whose elements are district positive integers in the range of 0 to n^2-1

- Matrix shows four and nine intensity levels

$$D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \qquad D_3 = \begin{bmatrix} 7 & 2 & 6 \\ 4 & 0 & 1 \\ 3 & 8 & 5 \end{bmatrix}$$

- Explain Halftone shading technique and compare it with dithering technique. 10M
- Write a note on Halftone shading technique.
- Write a note on dithering technique

Fractals

Topological dimension

- If the object can be deformed into the line we assign its dimension Dt=1
- If the object can be deformed into the plane or half plane or disk we assign its dimension Dt=2
- If the object can be deformed into the space or half space or sphere we assign its dimension Dt=3

The dimension Dt is referred as topological dimension

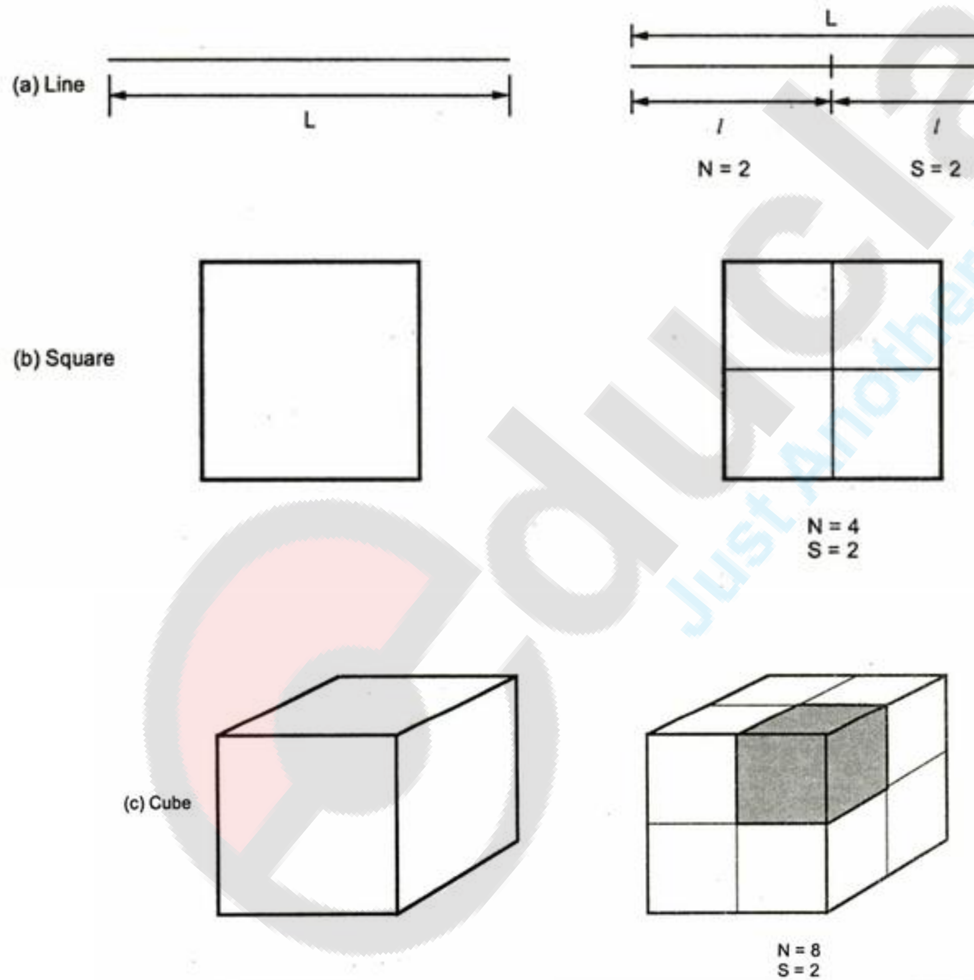
Fractal dimension

- Object dimension can be measured using fractal dimension.
- Imagine that the line segment of length L is divided into N identical pieces ,the length of each line segment can be given as l

  $$l = L/N$$

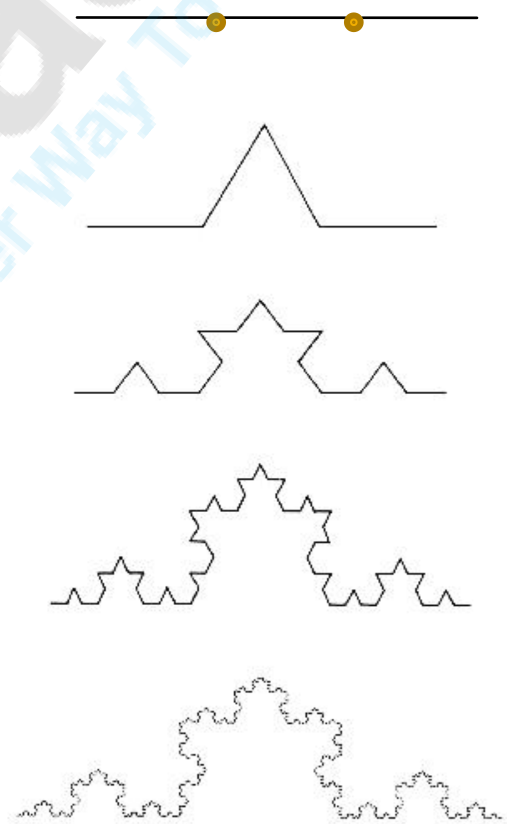
- Scaling factor: ratio of length of original line segment (L) and the length of each part of line segment(l) is called scaling factor.

  $$S = L/l$$


(a) Line
L
L
l
l
N = 2
S = 2
(b) Square
N = 4
S = 2
(c) Cube
N = 8
S = 2

Koch curve

- It can be drawn by dividing line into 3 equal segments with scaling factor 1/3 and replace the middle one by two pieces like a tent The same is done for all 4 sections .

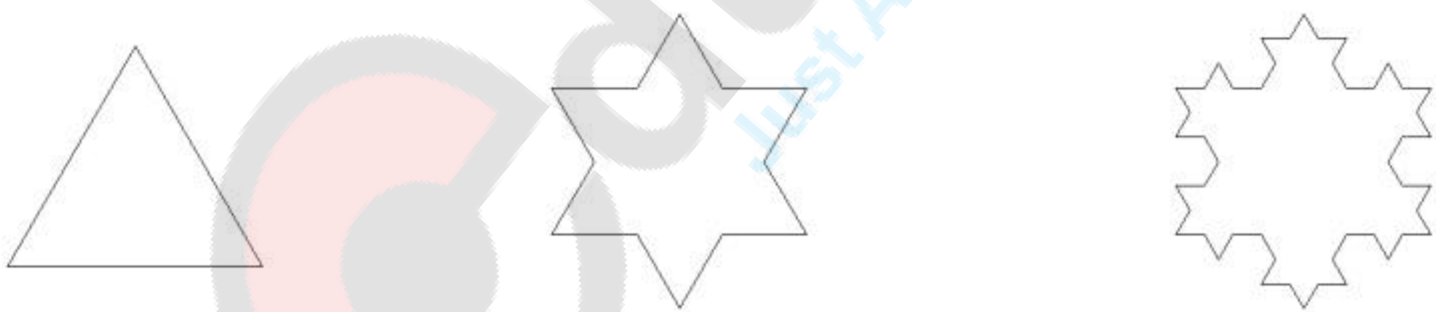


properties

- Each repetition increases the length of the curve by factor 4/3
- Length of the curve is infinite.
- It doesn't much deviate from its original shape
- Koch curve topological dimension is 1 and fractal dimension is 1.2618

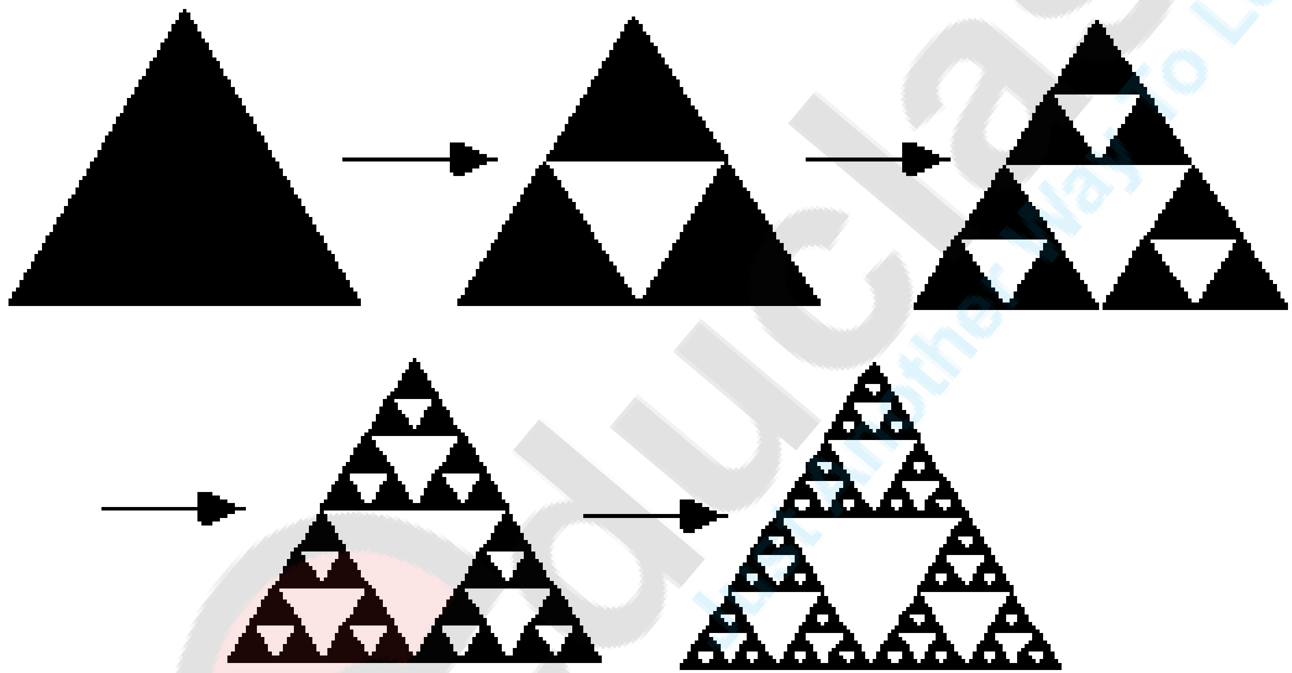
Koch Snowflake

- Basically the Koch Snowflake are just three Koch curves combined to a regular triangle. The construction rules are the same as the ones of the Koch curve.



Sierpinski triangle

- Take a triangle
- Create four triangles out of that one by connecting the centers of each side
- Cut out the middle triangle
- Repeat the process with the remaining triangles
- The number of triangles in the Sierpinski triangle can be calculated with the formula:
- Where n is the number of triangles and k is the number of iterations.



- What is fractal ? List and explain different types of fractals? 10M
- Explain Koch curve? 5M
- What is fractal ? What are its different types? how is a fractal dimension measures ? 10m