MCA (SEM- II)
Probability and Statistics
(OCT-16)

Q.P. Code : 510902

(3 Hours) **[Total Marks : 100**

N.B
(1) Question No1 is compulsory.
(2) Attempt any four questions out of remaining six questions.
(3) Assume necessary data but justify the same
(4) Figures to the right indicate full marks
(5) Use of scientific calculator is allowed

Q1. (a) [10]

Age in years	20-25	25-30	30-35	35-40	40-45	45-50
No. of policy holders	2	7	5	2	4	5

For the above frequency distribution Find
i) Inter Quartile Range
ii) Quartile Deviation and its coefficient
iii) Bowley's coefficient of skewness
iv) Range and its coefficient

Q1. (b) What is the probability that all vowels come together in the word "COMMERCE"? [05]

Q1. (c) Prove that geometric distribution is memory less [05]

Q2. (a) The joint probability density function of two dimensional random variable (X, Y) is given by [08]

$f(x,y) = 2-x-y \quad 0 \le x \le 1,\ 0 \le y \le 1$
$\quad = 0, \quad$ otherwise

i) Find marginal density functions of x and y.
ii) Find the conditional density function of $x|y$ and $y|x$.
iii) Find Expectation of (x) and Expectation(Y)

Q2. (b) If X and Y are independent Poisson variates show that the conditional distribution of X given X+Y is binomial [07]

Q2. (c) Theory predicts that the proportion of beans, in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1,600 beans, the number in the four groups was 882, 313, 287 and 118. Does the experimental result support the theory? [05]
(tabulated value for $\chi 2$ for 3 d.f. at 5% LOS is 7.81)

Q3. (a) Find the Karl Pearson's skewness coefficient for the following data [08]

Class Interval	0 - 10	10 – 20	20 - 30	30 - 40	40 - 50
frequency	11	25	61	93	70

Q3. (b) In manufacturing a certain component, two types of defects are likely to occur with respective probabilities 0.05 and 0.1. what is the probability that a randomly chosen component [07]

i) does not have either kind of defects
ii) Is defective?
iii) Has one kind of defect, given that it is found to be defective?

Q3. (c) Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 4 trains, find [05]

i) probability that the yard is empty
ii) The average number of trains in the system

Q4. (a) In an experiment of tossing of four coins, if X denotes 'number of heads, find [08]

i) The probability mass function of X,
ii) The distribution function of X
iii) P(X<=3)
iv) Variance of X

Q4. (b) The height of a group of 1000 students follows a normal distribution with mean 165cm and standard deviation 5 cm. Find the number of students having height [07]

(i) up to 171 cm,
(ii) below 165 cm
(iii) between 160 to 170

[Given P(0<=Z<=1.2)=0.3849, P(0<=Z<=1)=0.3413 where Z is a standard normal variate]

Q4. (c) A machine is designed to produce insulation washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024 cm with a standard deviation of 0.002 cm. Test the significance of the deviation. Value of t for 9 degrees of freedom at 5% level is 2.262 [05]

Q5. (a) A committee of 4 persons is to be appointed from 3 officers of the production department, 4 officers of purchase department, 2 officers of sales department and 1 charted accountant. Find the probabilities of forming the committee in the following manner. [08]

i) There must be one from each category
ii) There must be at least one person from purchase department

Q5. (b) In a sample of 12 fathers and their eldest sons gave the following data about their height in inches. [07]

Father	65	63	67	64	68	62	70	66	68	67	69	71
Son	68	66	68	65	69	66	68	65	71	67	68	70

Calculate rank correlation coefficient.

Q5. (c) The letters of the word 'failure' are arranged at random. Find the probability that the consonants may occupy only odd positions [05]

Q6. (a) X and Y are two random variables having joint density function [08]

X\Y	0	1	2
0	1/12	1/6	1/12
1	1/24	1/12	1/24
2	1/8	1/4	1/8

i) Find the marginal Density functions of X and Y
ii) Find the conditional distribution of Y for X=x.
iii) Are X and Y independent?
iv) Are X and Y uncorrelated?

Q6. (b) Prove with example that three events may be mutually independent but need not pair wise independent. [07]

Q6. (c) The size, mean and standard deviation of three samples is shown in the table below Find the combined mean and combined standard deviation. [05]

Sample -->	Sample1	Sample2	Sample3
Sample size	75	150	25
Mean	20	25	30
Standard Deviation	5	7	6

Q7. (a) Suppose a random variable X takes on the values -3,-1,2 and 5 with probabilities [08]

x	-3	-1	2	5
p(x)	$\frac{2k-3}{10}$	$\frac{k-2}{10}$	$\frac{k-1}{10}$	$\frac{k+1}{10}$

i) Determine k and pmf of x
ii) Determine the distribution function of x
iii) Find the expected value of x
iv) Find the Variance of X
v)

Q7. (b) Find the mean and variance of beta distribution of first kind [07]

Q7. (c) Calculate Modal marks for the following. [05]

Marks	10-30	30-50	50-70	70-90	90-110	110-130
No of students	4	10	14	12	8	6

Q.1 A)

Age in years	**20-25**	**25-30**	**30-35**	**35-40**	**40-45**	**45-50**
No of policy holders	**2**	7	5	**2**	**4**	**5**

For the above frequency distribution find

i. **Inter Quartile range**
ii. **Quartile deviation and its coefficient**
iii. **Bowley's coefficient of skeweness**
iv. **Range and its coefficient**

Solution:

i) first arrange the numbers in ascending order

2,2,4,5,5,7

Inter Quartile=upper Quartile-lower Quartile

=2.5-1

=1.5

ii)

Q1=Value of (n/4)th item =Value of (60/4)th item = 15th itemQ1=Value of (n/4)th item =Value of (604)th item = 15th item
Q1Q1 lies in the class 10.25−10.7510.25−10.75
∴∴ Q1=l+hf(n4−c)Q1=l+hf(n4−c)
Where l=10.25l=10.25, h=0.5h=0.5, f=12f=12, n/4=15n/4=15 and c=7c=7
∴∴ Q1=10.25+0.512(15−7)=10.25+0.33=10.58Q1=10.25+0.512(15−7)=10.25+0.33=10.58
Q3=Value of (3n4)th item =Value of (3×604)th item = 45th item

iv)
range= largest value – smallest value
=7-2
=5
Coefficient= 7-2/7+2
=5/9
=0.555

Q.1 B) what is the probability that all vowels come to gether in word "COMMERCE"?

Solution:-

Forming words using the letters of the word COMMERCE

In the given word

L = Total number of letters

= 8 {C, O, M, M, E, R, C, E}

La = Repeating letters of the first kind

= 2 {C's}

Lb = Repeating letters of the second kind

= 2 {M's}

Lc = Repeating letters of the third kind

= 2 {E's}

Lx = Letters without repetitions

= 2 {O, R}

L = La + Lb + Lc + Lx

Total number of Possible Choices

= Number of words that can be formed using the 8 letters of the word COMMERCE

⇒ n =

L!

La! × Lb! × Lc!

=

8!

2! × 2! × 2!

=

8 × 7 × 6 × 5 × 4 × 3 × 2!

2 × 1 × 2 × 1 × 2 × 1

= 8 × 7 × 6 × 5 × 3

= 5,040

Let,

A : The event of the word formed with all the vowels coming together

• For Event "A"

For the word to be formed taking the vowels as a unit

LG = Total number of letters

= 6 {C, M, M, R, (O, E, E), C}

LGa = Repeating letters of the first kind

= 2 {C's}

LGb = Repeating letters of the second kind

= 2 {M's}

LGx = Letters without repetitions

= 2 {R, (O, E, E)}

LG = LGa + LGb + LGx

For the first group (vowels)

LG1 = Total number of letters

= 3 {O, E, E}

LG1a = Repeating letters of the first kind

= 2 {E's}

LG1x = Letters without repetitions

= 1 {O}

LG1 = LG1a + LG1x

Number of favorable/favourable choices

= Number of ways in which a word can be formed (i.e. the no. of words that can be formed) using the letters of the word COMMERCE with all the vowels together.

⇒ mA = (Number of ways in which the 6 letters (taking the vowels as a unit) can be arranged) × (Number of ways in which the vowels can be interarragned between themselves)

=

LG!

LGa! × LGb!

×

LG1!

LG1a!

=

6!

2! × 2!

×

3!

2!

=

6 × 5 × 4 × 3 × 2!

2! × 2 × 1

×

3 × 2!

2!

= 180 × 3

= 540

Probability of forming words using all the letters of the word COMMERCE with all the vowels together

⇒ Probability of occurrence of Event 'A'

=

Number of favourable/favorable choices for the Event

Total number of possible choices for the Experiment

⇒ P(A) =

mA

n

=

540

5,040

=

3

28

Odds

Number of unfavorable choices

= Total number of possible choices - Number of favorable choices

⇒ mAc = n - mA

= 5,040 - 540

= 4,500

in favor/favour

Odds in Favour of forming words using all the letters of the word COMMERCE with all the vowels together

⇒ Odds in favor/favour of Event 'A'

= Number of favourable choices : Number of unfavorable choices

= m : mAc

= 540 : 4,500

= 3 : 25

against

Odds in Favour of forming words using all the letters of the word COMMERCE with all the vowels together

⇒ Odds against Event 'A'

= Number of unfavourable choices : Number of favorable choices

= mAc : m

= 4,500 : 540 = 25 : 3

Q.1 C) Prove that geometric distribution is memory less.

Ans-

Theorem The geometric distribution has the memoryless (forgetfulness) property.
Proof A geometric random variable X has the memoryless property if for all nonnegative integers s and t,
P(X _ s + t | X _ t) = P(X _ s)
or, equivalently
P(X _ s + t) = P(X _ s)P(X _ t).
The probability mass function for a geometric random variable X is
f(x) = p(1 − p)x x = 0, 1, 2, ...
The probability that X is greater than or equal to x is
P(X _ x) = (1 − p)x x = 0, 1, 2, ...
So the conditional probability of interest is
P(X _ s + t |X _ t) =
P(X _ s + t, X _ t)
P(X _ t)
=
P(X _ s + t)
P(X _ t)
=
(1 − p)s+t
(1 − p)t
= (1 − p)s
= P(X _ s),
which proves the memoryless property.
APPL verification: The APPL statements
simplify((1 - op(CDF(GeometricRV(p)))(s)[1]) * (1 - op(CDF(GeometricRV(p))(t)[1])));
1 - simplify(op(CDF(GeometricRV(p))(s + t)[1]));
both yield the expression
(1 − p)s+t.
1

Q2)If X and Y are independent Poisson variates show that the conditional distribution of X given X+Y is binomial.

Soln:- Let x & y be independent Poisson values variates with parameters λ_1 & λ_2 respectively.

Then X+Y is also Poisson variate with parameter $\lambda_1+\lambda_2$.

$$P[X=r \mid X+Y=n] = \frac{P[X=r \cap X+Y=n]}{P(X+Y=n)}$$

$$= \frac{P[X=r \cap Y=n-r]}{P(X+Y=n)}$$

$$= \frac{P[X=r] \times P[Y=n-r]}{P(X+Y=n)} \quad \because X \,\&\, Y \text{ are independent}$$

$$= \frac{\dfrac{e^{-\lambda_1}\lambda_1^{r}}{r!} \times \dfrac{e^{-\lambda_2}\lambda_2^{n-r}}{r!}}{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^{r}}$$

$$= \frac{n!}{r!(n-r)!} \times \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{r} \times \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-r}$$

$$= {}^{n}C_{r}\,p^{r}q^{n-r} \quad \text{where } p=\frac{\lambda_1}{\lambda_1+\lambda_2},\ q=1-p.$$

Hence conditional distribution of X given X+Y=n is binomial distribution with parameter n and $p=\frac{\lambda_1}{\lambda_1+\lambda_2}$

Q3)

Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 4 trains, find

i) **probability that the yard is empty**

ii) **The average number of trains in the system**

Soln:- $\lambda = 1/15$ per minute $\mu = 1/33$ per min.

$\rho = \lambda/\mu = 33/15 = 2.2$ $N = 4.$

(i) $P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{\rho-1}{\rho^{N+1}-1} = \frac{2.2-1}{2.2^5-1} = \frac{1.2}{51.5-1} = 0.0237$

(ii) Average no of trains in the system.

$L_s = \sum_{i=0}^{N} nP_i = 0 + P_1 + 2P_2 + 3P_3 + 4P_4$

$= P_0(\rho + 2\rho^2 + 3\rho^3 + 4\rho^4)$

$= 0.0237[2.2 + 2 \times 2.2^2 + 3 \times 2.2^3 + 4 \times 2.2^4]$

$= 0.0237[2.2 + 9.68 + 31.94 + 93.70]$

$= 3.26.$

Q4 a) In an experiment of tossing of four coins, if X denotes number of heads, find

i) The probability mass function of X

ii) The distribution function of X

iii) P(x<=3)

iv) Variance of X

Solution:

Four coins are tossed, X denotes the number of heads

S ={HHHH, HHHT, HHTH, HTHH, HHTT, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

⇒ The values carried by the variable ("x") would be either 0, 1, 2, 3 or 4

⇒ "X" is a discrete random variable with range = {0, 1, 2, 3, 4}

In the experiment of tossing an unbiased coin four times

Total no. of possible choices	=	$2 \times 2 \times 2 \times 2$
i)	=	16

⇒ P(No Heads) = 1/16

For the two letter word "TTTT"

No. of letters

in total = 5 ⇒ n = 4

of the first kind = 4 {T, T, T, T} ⇒ a = 4

all of which are different = 0 ⇒ x = 0

No. of favorable choices	=	No. of ways in which the two letters of the word TTTT can be permuted.

= n!/a!

= 4!/4! = 1

- One Heads = No. of favourable choices
  Total no. of possible choices

⇒ P(One Head) = 4/16

For the four letter word "HTTT"

No. of letters

in total = 4 ⇒ n = 4

of the first kind = 3 {T, T, T} ⇒ a = 3

all of which are different = 1 {H} ⇒ x = 1

No. of favorable choices	=	No. of ways in which the four letters of the word HTTT can be permuted.

= n!/a!

= 4!/3! = 4

- Two Heads = No. of favourable choices
  Total no. of possible choices

⇒ P(Two Head) = 6/16
For the four letter word "HHTT"
No. of letters
in total = 4 ⇒ n = 4
of the first kind = 2 {T, T} ⇒ a = 2
of the second kind = 2 {H, H} ⇒ b = 2
all of which are different = 0 {} ⇒ x = 0

No. of favorable choices	=	= No. of ways in which the four letters of the word HHTT can be permuted.

= n!/a! x b!
= 4!/2! x 2!
= 4 × 3 × 2! / 2 × 1 × 2! = 6

- Three Heads = No. of favourable choices
  Total no. of possible choices
  ⇒ P(Three Head) = 4/16
  For the four letter word "HHHT"
  No. of letters
  in total = 4 ⇒ n = 4
  of the first kind = 3 {H, H, H} ⇒ a = 3
  all of which are different = 1 {T} ⇒ x = 1

No. of favourable choices = No. of ways in which the four letters of the word HHHT can be permuted

= n! / a!
= 4! / 3!
= 4

- FourHeads = No. of favourable choices
  Total no. of possible choices

  ⇒ P(Two Head) = 1/16
  For the four letter word "HHHH"
  No. of letters
  in total = 4 cn = 4
  of the first kind = 4 {H, H, H, H} ⇒ a = 4
  all of which are different = 0 {} ⇒ x = 0

No. of favorable choices	=	= No. of ways in which the four letters of the word HHHH can be permuted.

= n!/a!
= 4!/4!
= 4

Therefore, probability that the no. of heads would be

- 0 ⇒ P(X=0) = 1/16
- 1 ⇒ P(X=1) = 4/16 OR 1/4
- 2 ⇒ P(X=2) = 6/16 OR 3/8
- 3 ⇒ P(X=3) = 4/16 OR 1/4
- 4 ⇒ P(X=4) = 1/16

ii) The" would be:

x	0	1	2	3	4
P(X = x)	1/16	4/16	6/16	4/16	1/16

iv)

x	x2	px2 [x2 x P(X = x)]
0	0	0
1	1	4/16
2	4	24/16
3	9	36/16
4	16	16/16
Total		80/16 = 5

Expected no. of Heads

⇒ Expectation of "x"

$\Rightarrow E(x) = \Sigma px$
$= 2$

Variance of the number of heads

$\Rightarrow var(x) = E(x^2) - (E(x))^2$

$\Rightarrow var(x) = \Sigma px^2 - (\Sigma px)^2$

$= 5 - (2)^2$

$= 5 - 4$

$= 1$

Q4. (c) A machine is designed to produce insulation washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024 cm with a standard deviation of 0.002 cm. Test the significance of the deviation. Value of t for 9 degrees of freedom at 5% level is 2.262

Solution:

Size of the given sample = 10 (small sample)

Sample mean = $\bar{x} = 0.024$ cm

Population mean = $\mu = 0.025$ cm

Degrees of Freedom = n-1 = 10 - 1 = 9

Table value of t for 9 df at 5% level of significance = 2.262

Null hypothesis H_0: The difference between $\bar{x}$ and μ is not significant.

Alternative hypothesis H_1: $\mu \neq 0.025$

The test statistic

$$t = \frac{\bar{x} - u}{\left(\frac{s}{\sqrt{n-1}}\right)} = \frac{0.024 - 0.025}{\frac{0.002}{\sqrt{10-1}}}$$

$$= \frac{-0.01}{0.02} \times 3 = -1.5$$

$$\therefore |t| = 1.5 < 2.262$$

Calculated value of t is less than the table value of t at 5% level of significance and 9 df. We accept null hypothesis and conclude that the difference between $\bar{x}$ and μ is not significant.

Q5. (a) A committee of 4 persons is to be appointed from 3 officers of the production department, 4 officers of purchase department, 2 officers of sales department and 1 charted accountant. Find the probabilities of forming the committee in the following manner.

i) There must be one from each category

ii) There must be at least one person from purchase department

Solution:

N(s) = 13

i) Probability that there must be one person from each category

$$= \frac{3C1 + 4C1 + 2C1 + 1C1}{10C4}$$

$$= \frac{\frac{3!}{2!} + \frac{4!}{3!} + \frac{2!}{1!} + \frac{1!}{1!}}{\frac{10!}{4!\,6!}} = 10/210 = 1/21 = 0.04761$$

i) Probability that there must be at least one person from purchase department

$$= \frac{4C1 + 4C2 + 4C3 + 4C4}{10C4}$$

$$= \frac{\frac{4!}{3!} + \frac{4!}{2!2!} + \frac{4!}{3!} + \frac{4!}{4!0!}}{\frac{10!}{4!\,6!}} = 14/210 = 1/15 = 0.667$$

Q5. (b) In a sample of 12 fathers and their eldest sons gave the following data about their height in inches.

Father	65	63	67	64	68	62	70	66	68	67	69	71
Son	68	66	68	65	69	66	68	65	71	67	68	70

Calculate rank correlation coefficient.

Solution:

Father	Son	Fr	Sr	d	di^2
65	68	9	5.5	3.5	12.25
63	66	11	9.5	1.5	2.25
67	68	6.5	5.5	1	1
64	65	10	11.5	-1.5	2.25
68	69	4.5	3	1.5	2.25
62	66	12	9.5	2.5	6.25
70	68	2	5.5	-3.5	12.25
66	65	8	11.5	-3.5	12.25
68	71	4.5	1	3.5	12.25
67	67	6.5	8	-1.5	2.25
69	68	3	5.5	-2.5	6.25
71	70	1	2	-1	1
Total					72.5

$= 1 - 6 * 72.5 / 12(12^2 - 1)$
$= 1 - 435 / 1716$
$= 1 - 0.2535$
$= 0.7465$

Q5 c) The letters of the word 'failure' are arranged at random. Find the probability that the consonants may occur only in odd positions.
Solution:

The word Failure has 4 Vowels and 3 Consonants
If we consider 3 consonants to be placed at 4 odd places i.e. 1^{st}, 3^{rd}, 5^{th}, 7^{th} then the probability is ${}^4C_3$
The probability of 3 consonants is 3! and of 4 vowels is 4!

Therefore,
= ${}^4C_3$ * 3! * 4!
= 4!/3! * 3! * 4!
= 4! * 4!
= 576

Q6. (c) The size, mean and standard deviation of three samples is shown in the table below Find the combined mean and combined standard deviation.

Sample -->	**Sample1**	**Sample2**	**Sample3**
Sample size	75	150	25
Mean	20	25	30
Standard Deviation	5	7	6

Solution:-

Find combined mean from the following data

$N_1 = 75$, $\bar{X}_1 = 20$
$N_2 = 150$, $\bar{X}_2 = 25$
$N_3 = 25$, $\bar{X}_3 = 30$

Solution:-
Combined mean:

$$\bar{X}_{12} = \frac{N_1 \cdot \bar{X}_1 + N_2 \cdot \bar{X}_2 + N_3 \cdot \bar{X}_3}{N_1 + N_2 + N_3}$$

$$\bar{X}_{12} = \frac{75 \cdot 20 + 150 \cdot 25 + 25 \cdot 30}{75 + 150 + 25}$$

$$\bar{X}_{12} = \frac{1500 + 3750 + 750}{250}$$

$$\bar{X}_{12} = \frac{6000}{250}$$

$$\bar{X}_{12} = 24$$

Find standard deviation from the following data

$N_1 = 75,\ \bar{x}_1 = 20,\ \sigma_1 = 5$

$N2 = 150,\ \bar{x}_2 = 25,\ \sigma_2 = 7$

$N3 = 25,\ \bar{x}_3 = 30,\ \sigma_3 = 6$

Solution:

Combined mean

$$\bar{x}_{12} = \frac{N_1 \cdot \bar{x}_1 + N_2 \cdot \bar{x}_2 + N_3 \cdot \bar{x}_3}{N_1 + N2 + N3}$$

$$\bar{x}_{12} = \frac{75.20 + 150.25 + 25.30}{75 + 150 + 25}$$

$$\bar{x}_{12} = \frac{1500 + 3750 + 750}{250}$$

$$\bar{x}_{12} = \frac{6000}{250}$$

$$\bar{x}_{12} = 24$$

Combined mean = 24

$d_1 = \bar{x}_1 - \bar{x}_{12} = 20 - 24 = -4$

$d2 = \bar{x}_2 - \bar{x}_{12} = 25 - 24 = 1$

$d3 = \bar{x}_3 - \bar{x}_{12} = 30 - 24 = 6$

Combined standard deviation

$$\sigma_{12} = \sqrt{\frac{N_1 \cdot (\sigma_1^2 + d_1^2) + N_2 \cdot (\sigma_2^2 + d_2^2) + N_3 \cdot (\sigma_3^2 + d_3^2)}{N_1 + N2 + N3}}$$

$$\sigma_{12} = \sqrt{\frac{75.(25+16) + 150.(49+1) + 25.(36+36)}{75 + 150 + 25}}$$

$$\sigma_{12} = \sqrt{\frac{3075 + 7500 + 1800}{250}} = \sqrt{\frac{12375}{250}}$$

$$\sigma_{12} = 7.04$$

Combined SD = 7.04

Q7. (c) Calculate Modal marks for the following.

Marks	10-30	30-50	50-70	70-90	90-110	110-130
No of students	4	10	14	12	8	6

Solution:

From the above table,

$F_0 = 14$, $F_1 = 10$, $F_2 = 12$, $h=20$

Modal class is 50-70

Mode $= L_1 + [(F_0 - F_1) / 2F_0 - F_1 - F_2] * h$

$= 50 + [(14-10) / 2*14-10-12] * 20$

$= 50 + (4/6) * 20$

$= 50 + 13.33$

$= 63.33$