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Q.1 Determine weather a sequence a_n is the solution of a recourrence relation.

$a_n = 2a_{n-1} - a_{n-2}$ for $n = 2, 3, ---$

n. $a_n = 5$ for every non negative integer n.

→

$a_n = 5$

$a_{n-1} = 5$

$a_{n-2} = 5$.

$5 = 2 \times 5 - 5$. $\therefore 5 = 5$

L.H.S = R.H.S

$\therefore$ Given relation is recourrence relation.

Q.2 Test weather a_n is a solⁿ of the recurrence relation,

$a_n = 2a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, ---$

$a_n = 2n$ for every non -ve integer.

→

$a_n = 2a_{n-1} - a_{n-2}$ —(1)

Given, $a_n = 2n$

$a_{n-1} = 2(n-1)$

$a_{n-2} = 2(n-2)$

~~$a_n = 2n$~~

Q.1

L.H.S : $a_n = 2n$

R.H.S : $2a_{n-1} - a_{n-2}$

$= \cancel{2\times(}\, 2\times 2(n-1) - 2(n-2)$

$= 4n - 4 - 2n + 4$

$= 2n \quad = \text{L.H.S.}$

$\therefore a_n = 2n$ is a solⁿ of given difference eqⁿ.

Q.1 # Solution of Recurrence relation.

$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} = f(n)$$

is $3^n + 4^n + 2$ $\quad f(n) = 6$ for all n.

determine c_0, c_1 & c_2.

→

Given,

The solution is

$$a_n = 3^n + 4^n + 2$$

$$\therefore a_{n-1} = 3^{n-1} + 4^{n-1} + 2^{\cancel{n-1}}$$

$$a_{n-2} = 3^{n-2} + 4^{n-2} + 2$$

$$f(n) = 6$$

$$\underline{C_1 \cdot 3^{n-1}} = C_1 \cdot 3^n \cdot 3^{-1} = \frac{C_1 \cdot 3^n}{3}$$

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$$C_0\left[3^n + 4^n + 2\right] + C_1\left[3^{n-1} + 4^{n-1} + 2\right]$$

$$+ C_2\left[3^{n-2} + 4^{n-2} + 2\right] = 6.$$

Collecting the coefficient of 3^n & 4^n and constant at both side we get.

$$C_0 3^n + C_0 \cdot 4^n + 2C_0 + C_1 3^{n-1} + C_1 \cdot 4^{n-1} + 2 \cdot C_1$$

$$+ C_2 \cdot 3^{n-2} + C_2 \cdot 4^{n-2} + 2 \cdot C_2 = 6.$$

$$3^n\left(C_0 + \frac{C_1}{3} + \frac{C_2}{3^2}\right) + 4^n\left(C_0 + \frac{C_1}{4} + \frac{C_2}{4^2}\right)$$

$$+ 2\left(C_0 + C_1 + C_2\right) = 0 \times 3^n + 0 \times 4^n + 6.$$

Comparing coeffient of L.H.S & R.H.S. for 3^n, 4^n & constant term. we get 3 eqn.

$$C_0 + \frac{C_1}{3} + \frac{C_2}{3^2} = 0 \quad -(2)$$

$$C_0 + \frac{C_1}{4} + \frac{C_2}{4^2} = 0 \quad -(3)$$

$$2\left(C_0 + C_1 + C_2\right) = 6 \quad -(4).$$

$$C_0 + C_1 + C_2 = 3.$$

$$C_0 = 3 - C_1 - C_2$$

From (2),

$$(3 - c_1 - c_2) + \frac{c_1}{3} + \frac{c_2}{9} = 0 \quad \text{---} (5)$$

From (3),

$$(3 - c_1 - c_2) + \frac{c_1}{4} + \frac{c_2}{16} = 0 \quad \text{---} (6)$$

From (5), L.C.M

$$\frac{9(3 - c_1 - c_2) + 3c_1 + c_2}{9} = 0$$

$$27 - 9c_1 - 9c_2 + 3c_1 + c_2 = 0$$

$$-6c_1 - 8c_2 + 27 = 0 \quad \text{---} (7)$$

From (6),

$$\frac{16(3 - c_1 - c_2) + 4c_1 + c_2}{16} = 0$$

$$48 - 16c_1 - 16c_2 + 4c_1 + c_2 = 0$$

$$-12c_1 - 15c_2 + 48 = 0 \quad \text{---} (8)$$

3 by 3 determinent.

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-48
27
21

$-12C_1 - 15C_2 + 48 = 0$

$-6C_1 - 8C_2 + 27 = 0 \quad \times 2.$

$-12\cancel{C_1} - 15C_2 + 48 = 0.$

$- \quad -1\cancel{2}C_1 - 16C_2 + 54 = 0$

$\quad + \quad + \quad -$

$C_2 - 6 = 0.$

$\boxed{C_2 = 6}$

$-6 \times C_1 - 8 \times 6 + 27 = 0.$

$-6C_1 - 48 + 27 = 0.$

$-6C_1 - 21 = 0.$

$-6C_1 = 21.$

$C_1 = \frac{21}{6} = \frac{-7}{2} //$

$\boxed{C_1 = -7/2}$

From (4),

$C_0 + C_1 + C_2 = 3.$

$C_0 = 3 - C_1 - C_2$

$= 3 + \frac{7}{2} - 6$

$= \frac{6 + 7 - 12}{2}$

$\boxed{C_0 = \frac{1}{2}}$

Sol^n

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$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} = F(n)$$

Sol^n s : $a_n = 2^n + 3^n + 5$

$F(n) = 40 \quad \forall n$

Find c_1, c_2, c_3.

$\rightarrow$

$$c_0(2^n + 3^n + 5) + c_1(2^{n-1} + 3^{n-1} + 5) + c_2(2^{n-2} + 3^{n-2} + 5) = 40.$$

$$2^n\left(c_0 + \frac{c_1}{2} + \frac{c_2}{2^2}\right) + 3^n\left(c_0 + \frac{c_1}{3} + \frac{c_2}{3^2}\right) + 5(c_0 + c_1 + c_2) = 40.$$

$$c_0 + \frac{c_1}{2} + \frac{c_2}{4} = 0. \quad -(1)$$

$$c_0 + \frac{c_1}{3} + \frac{c_2}{3^2} = 0. \quad -(2)$$

$$c_0 + c_1 + c_2 = 8. \quad -(3)$$

$4 \quad -20 \quad +24$

$4 + 4$

(8)

$$\Delta C_1 = \begin{vmatrix} 0 & 1/2 & 1/4 \\ 0 & 1/3 & 1/9 \\ 8 & 1 & 1 \end{vmatrix}$$

2/9

$$\Delta = \begin{vmatrix} 1 & 1/2 & 1/4 \\ 1 & 1/3 & 1/9 \\ 1 & 1 & 1 \end{vmatrix}$$

$-1/18$

$$\Delta = 1\left(\frac{1}{3} - \frac{1}{9}\right) - \frac{1}{2}\left(1 - \frac{1}{9}\right) + \frac{1}{4}\left(1 - \frac{1}{3}\right)$$

$$= 1\left(\frac{3-1}{9}\right) - \frac{1}{2}\left(\frac{9-1}{9}\right) + \frac{1}{4}\left(\frac{3-1}{3}\right)$$

$$= 1\left(\frac{2}{9}\right) - \frac{1}{2}\left(\frac{8}{9}\right) + \frac{1}{4}\left(\frac{2}{3}\right)$$

$$= \frac{2}{9} - \frac{8}{18} + \frac{2}{12}$$

$$= \frac{8 - 16 + 6}{36} = \frac{-8+6}{36} = \frac{-\cancel{2}^{1}}{\cancel{36}_{18}} = \frac{-1}{18}$$

$$\Delta C_1 = \begin{vmatrix} 0 & 1/2 & 1/4 \\ 0 & 1/3 & 1/9 \\ 8 & 1 & 1 \end{vmatrix}$$

$$= 0 - \frac{1}{\cancel{2}_1}\left(0 - \frac{\cancel{8}^4}{9}\right) + \frac{1}{\cancel{4}_1}\left(-\frac{\cancel{8}^2}{3}\right)$$

$$= \frac{4}{9} - \frac{2}{3}$$

$$\frac{4-6}{9}$$

$$\frac{-2}{9}$$

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(9)

$-1/18$

$$\Delta C_2 = \begin{vmatrix} 1 & 0 & 1/4 \\ 1 & 0 & 1/9 \\ 1 & 8 & 1 \end{vmatrix}$$

$$= 1\left(\frac{-8}{9}\right) + \frac{1}{\cancel{4}_1}\left(\overset{2}{\cancel{8}}\right)$$

$$= \frac{-8}{9} + \frac{2}{\times 9}$$

$$= \frac{-8+18}{9}$$

$$= \frac{10}{9}$$

$$\Delta C_3 = \begin{vmatrix} 1 & 1/2 & 0 \\ 1 & 1/3 & 0 \\ 1 & 1 & 8 \end{vmatrix}$$

$\frac{-1}{18}$

$$= 1\left(\frac{8}{3} - 0\right) - \frac{1}{2}(8-0) + 0$$

$$= 1\left(\frac{8}{3}\right) - \frac{1}{\cancel{2}_1}\left(\overset{4}{\cancel{8}}\right)$$

$$= \frac{8}{3} - \frac{4}{\times 3}$$

$$= \frac{8-12}{3} = \frac{-4}{3}$$

$$\frac{\Delta C_1}{\Delta} = \frac{-2}{\cancel{9}_1} \times -\cancel{18}^{2} = 4$$

$$\frac{\Delta C_2}{\Delta} = \frac{10}{9} \times -18^{2} = (-20)$$

$$\frac{\Delta C_3}{\Delta} = \frac{-4}{3} \times -18^{6} = (24)$$

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Recurrence Relation.

Use back tracking method to find the sequence $\{a_n\}$ as a solution of the recurrence relation,

$$a_n = a_{n-1} + 3 , \quad a_1 = 2.$$

$$a_n = a_{n-1} + 3 \quad \text{--- (1)}$$

Put $n = n-1$ in eqⁿ (1)

$$a_{n-1} = a_{n-2} + 3 \quad \text{--- (2)}$$

eqⁿ (I) becomes,

$$a_n = [a_{n-2} + 3] + 3.$$

$$a_n = a_{n-2} + 2 * 3 \quad \text{--- (3)}.$$

If we put $n=n-1$ in eqⁿ (2), we get.

$$a_{n-2} = a_{n-3} + 3.$$

we use it in eqⁿ (3),

$$a_n = [a_{n-3} + 3] + 3 + 3.$$

$$a_n = a_{n-3} + 3 + 3 + 3.$$

$$a_n = a_{n-3} + 3 \times (3)$$

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$$a_n = a_{n-k} + k \cdot 3.$$

Substitute $k = n-1$ in the above eqn, we get,

$$a_n = a_{n-(n-1)} + (n-1) \times 3.$$

$$a_n = a_1 + 3(n-1).$$

$$a_n = 3(n-1) + 2.$$

These is the solution for given ~~as~~ recurrence solution.

Q.2 Find the soln of recurrence relation
$b_n = 2b_{n-1} + 1$, $b_1 = 7$.

→

$$b_n = 2b_{n-1} + 1 \quad \text{—— (1)}$$

$$b_{n-1} = 2b_{n-2} + 1 \quad \text{—— (2)}$$

$$b_n = 2[2b_{n-2} + 1] + 1$$

$$b_n = 2^2 b_{n-2} + 2 + 1 \quad \text{—— (3)}$$

$$b_n = 2^2 [2b_{n-3} + 1] \; 2 + 1$$

$$b_n = 2^3 \cdot b_{n-3} + 2^2 + 2 + 1.$$

$$b_n = 2^k \cdot b_{n-k} + 2^{k-1} + 2^{k-2} \cdots + 2 + 1$$

Put $\underline{k = n-1}$

$$b_n = 2^{n-1} b_{n-(n-1)} + 2^{n-2} + 2^{n-3} \dots + 2 + 1.$$

$$b_n = 2^{n-1} b_1 + \boxed{2^{n-2} + 2^{n-3} + \dots + 2 + 1}$$

$$a + ar + ar^2 + \dots + ar^{n-1}$$

$$a\left[1 + r + r^2 + r^3 + \dots + r^{n-1}\right] = \frac{a\,[r^n - 1]}{r-1}$$

$$1 + r + r^2 \dots + r^{n-1} = \frac{r^n - 1}{r-1}$$

$$1 + 2 + 2^2 + \dots + 2^{n-2} = \frac{2^{n-1} - 1}{2-1}$$

$$b_n = 2^{n-1} \cdot b_1 + \frac{2^{n-1} - 1}{1}$$

$$b_n = 2^{n-1} \times 7 + 2^{n-1} - 1$$
$$= 8 \cdot 2^{n-1} - 1$$
$$b_n = 2^3 \cdot 2^{n-1} - 1$$
$$= 2^{n-1+3} - 1$$
$$b_n = 2^{n+2} - 1$$

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State the problems of towers of Hanoi. Find the recurrence relation and solve it.

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let n be the no. of disk

let a_n is the no. of moves required to move no. of disk.

For $n=1$	$a_n = 1$ ~~move~~	move	
	~~from~~ ~~to~~	from	to
n	a_n		
$n=2$	3.	A	C
		A	B
		C	B.

~~$n=1$~~	n	a_n	
	1	1	a_1
	2	3	$a_2 = 1+1+1 = 2a_1+1$
	3	7	$a_3 = 3+3+1 = a_2 + a_2 + 1 = 2a_2+1$
	4	15	$a_4 = 7+7+1 = a_3+a_3+1 = 2a_3+1$

For n disk we get.

$$n = 2a_{n-1} + 1$$

this the recurrence relⁿ for the tower of hanoi problem.

We can solve this by back tracking method.

$$a_n = 2a_{n-1} + 1 \quad \text{---(1)} \qquad a_1 = 1.$$

$$a_{n-1} = 2a_{n-2} + 1 \quad \text{---(2)}.$$

eqⁿ (1) become,

$$a_n = [2a_{n-2} + 1] + 1$$

$$a_n = 2^2 \{ a_{n-2} + 2 + 1$$

$$a_n = 2^2 [2a_{n-3} + 1] + 2 + 1$$

$$a_n = 2^3 a_{n-3} + 2^2 + 2 + 1$$

$$a_n = 2^k a_{n-k} + 2^{k-1} + 2^{k-2} - - + 1.$$

Put, $k = n - 1$

$$a_n = 2^{n-1} \cdot a_1 + 2^{n-1-1} + 2^{n-1-2} - - + 2 + 1$$

$$a_n = 2^{n-1} \times 1 + 2^{n-2} + 2^{n-3} \cdots + 2 + 1$$

$$a_n = 2^{n-1} + 2^{n-2} + \cdots + 2 + 1.$$

$$a_n = \frac{2^n - 1}{2 - 1}$$

$$\boxed{a_n = 2^n - 1}$$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

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Sr.	Nature of Charactristic Roots.	Homogeneous sol^n.
1.	Real & distinct m_1, m_2	$a_n = A(m_1)^n + B(m_2)^n$.
2.	Real & Repeated $m_1 = m_2 = m_3 = m$.	$a_n = (An^2 + Bn + C)(m)^n$
3.	Real, some are repeated & some are distinct $m_1 = m_2 = m, m_3$	$a_n = (An + B)(m)^n + C(m_3)^n$

Q. 1 Solve the Recurrence relation, $a_n = a_{n-1} + 2a_{n-2}$ with initial condition $a_0 = 2$, $a_1 = 7$.

→

as RHS of eq^n equal to zero we only have homogenuous eq^n.

$$a_n - a_{n-1} - 2a_{n-2} = 0 \quad \text{—— } 1$$

$$\left[\begin{matrix} 1 & -1 & -2 & = 0 \end{matrix}\right] \quad \text{write coef}^n\text{:}$$

$$m^2 - m - 2 = 0.$$

$$m_1 = 2 \quad \text{OR} \quad m = -1$$

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Real & distinct,

$a_n = A(2)^n + B(-1)^n$ —— (2).

Given initial conditions,

$a_0 = 2$.

We put $n=0$ in eq^n (2).

$a_0 = A(2)^0 + B(-1)^0$

$2 = A + B$ —— (3)

$a_1 = 7$.

We put $n=1$ in eq^n (2),

$a_1 = A(2)^1 + B(-1)^1$

$7 = 2A - B$ —— (4)

Solve eq^n (3) & (4),

$$A + \cancel{B} = 2$$
$$+ \; 2A - \cancel{B} = 7$$

$$3A = 9$$
$$\boxed{A = 3}$$

Put $A = 3$ in eq^n (3),

$$A + B = 2$$
$$3 + B = 2$$
$$\boxed{B = -1}$$

Put the value in eq^n (2),

$$a_n = 3(2)^n - (-1)^n$$

2) Find the solⁿ of Recurrence relⁿ $a_n = 3a_{n-1} - 2a_{n-2}$ with initial condⁿ $a_1 = 5$, $a_2 = 3$.

$\longrightarrow$

$$a_n - 3a_{n-1} - 2a_{n-2} = 0 \quad \text{--- (1)}$$

$$m^2 - 3m - 2 = 0$$

2
2 1

$$m^2 - 2m - 1m - 2 = 0$$
$$m(m-2) - 1(m-2) = 0$$
$$(m-1)(m-2) = 0$$
$$\underline{m_1 = 1} \quad \text{OR} \quad \underline{m_2 = 2}$$

Real & distinct.

$$a_n = A(m_1)^n + B(m_2)^n$$

$$= A(1)^n + B(2)^n \quad \text{--- (2)}$$

Given Initial condⁿ,

$a_1 = 5$.

$$a_1 = A(1)^1 + B(2)^1$$

$$a_1 = A + 2B$$
$$A + 2B = 5 \quad \text{--- (3)}$$

$a_2 = 3$

$$a_2 = A(1)^2 + B(2)^2$$

$$3 = A + 4B$$

$$A + 4B = 3 \quad —(4)$$

Solve eq^n (3) & (4),

$$A + 4B = 3$$
$$\underline{-\ A + 2B = 5}$$

$$2B = -2$$

$$\boxed{B = -1}$$

Put $B = -1$ in eq^n (3),

$$A + 2(-1) = 5.$$

$$A - 2 = 5$$

$$A = 5 + 2$$

$$\boxed{A = 7}$$

Put in eq^n, (2),

$$a_n = 7(1)^n + -1(2)^n.$$

3) Find the recurrence relation

$a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1$, $a_1 = 2$

→

$a_n - 6a_{n-1} + 9a_{n-2} = 0$ — (1)

$m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$\underline{m = 3}$ OR $\underline{m = 3}$

As roots are repeated,

$a_n = (An + B)(m)^n$

$a_n = (An + B)(3)^n$ — (2)

Given initial condⁿ,

$a_0 = 1$

$a_0 = (A0 + B)(3)^0$

$1 = B$ — (3)

Put $a_1 = 2$

$a_1 = (A1 + B)(3)^1$

$2 = (A + B)(3)$

$A + B = 6$

$\boxed{A + B = 6}$ — (4)

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(4) Solve the recurrence relalation,

$a_n + 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 0$.

$a_0 = 1, a_1 = -2, a_2 = 8$.

$\rightarrow$

$a_n + 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 0$.

$m^3 + 6m^2 + 12m + 8 = 0$. — (1).

$(m+2)(m^2 + 4m + 4) = 0$.

$(m+2)(m^2 + 2m + 2m + 4) = 0$.

$(m+2)(m+2)(m+2) = 0$.

$m_1 = -2, m_2 = -2, m_3 = -2$.

Real & Reapeated,

$a_n = (An^2 + Bn + c)(m^n)$

$a_n = (An^2 + Bn + c)(-2)^n$. — (2)

$a_0 = 1$ — $\cancel{a_0}$ —

$1 = C(-2)^0$. — (3)

$a_1 = -2$ — $-2 = (A + B + c)(-2)^1$

$+1 = A + B + C$. — (4)

$a_2 = 8$ $\quad$ ~~$1 = A+B+C$~~ — (4)

$8 = (4A + 2B + C)(-2)^2$

$4A + 2B + C = 8/4$

$4A + 2B + C = 2$ — (5)

$C = 1$ — (8)

$A + B + C = 1$ — (9) $\times 2$

$4A + 2B + C = 2$ — (1)

$$\cancel{\begin{aligned} 2A + 2B + 2C &= 1 \\ 4A + 2B + C &= 2 \\ \hline -2A &= -1 \\ A &= \frac{-1}{2} \end{aligned}}$$

Put $C = 1$.

$A + B = 0$ — (6)

$4A + 2B = 1$ — (7)

$2A + 2B = 0$

$4A + 2B = 1$

$-2A = -1$

$\boxed{A = 1/2}$

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$\boxed{B = -1/2}$

$$a_n = \left(\frac{1}{2}n^2 - \frac{1}{2}n + 1\right)(-2)^n$$

To Find the particulars solution $(a_n^{(P)})$

R.H.S of eq^n not equal to zero, we have to see the type of R.H.S and then write the particular solution The rules are given table in below,

Form of R.H.S	Form of P.S.
1. Constant	$a_n = A$.
2. linear expression eg. $2n+5$	$a_n = An + B$
3. Quadratic expression eg. $3n^2 + 2n + 5$.	$a_n = An^2 + Bn + C$
4. constant $\times$ expo. 4^n, 42×4^n	$a_n = A \cdot 4^n$. Note: base of the expression is not

Total solution :-
Homogenous + Particular

(5) linear expression $a_n = (An+B)4^n$.
eg. $(2n+5)(4^n)$

(6) Quad. expn. $a_n = (An^2+Bn+C)4^n$.

$(3n^2+2n+5) \times 4^n$.

(1) Find the particular solⁿ of the recurrence relation.

$$a_n + 4a_{n-1} + 4a_{n-2} = n^2 - 3n + 5 \quad \text{---(1)}$$

→ To find the particular solⁿ, consider the R.H.S.

$n^2 - 3n + 5$ - it is quadratic eqⁿ.

∴ Particular solⁿ is given by.

$$a_n^{(P)} = An^2 + Bn + C.$$

$$a_{n-1}^{(P)} = A(n-1)^2 + B(n-1) + C$$

$$a_{n-2}^{(P)} = A(n-2)^2 + B(n-2)^2 + C.$$

Use them in eqⁿ (1),

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$\rightarrow (An^2 + Bn + C) + 4(A(n-1)^2 + B(n-1) + C)$

$+4(A(n-2)^2 + B(n-2) + C) = n^2 - 3n + 5$

$\rightarrow 4[A(n^2 - 2n + 1) + B(n-1) + C]$

$An^2 - 2An + A + Bn - B + C$

$4An^2 + n4(-2A + B) + 4(A - B + 4)$

$\rightarrow 4(A(n^2 - 4n + 4) + Bn - 2B + C]$

$4An^2 - 16n + 16 + Bn - 2B + C$

$4An^2 + n(-16 + B) + (16 - 2B + C)$

$n^2[A + 4A + 4A] + n[B + 4(-2A + B + (-16 + B))$

$+ C\{+4(A - B + C) + (16 - 2B + C)]$

$= n^2 - 3n + 5.$

$A = 1/9$

$B = -13/27.$

$C = 7/9.$

(2) Find the particular solⁿ of the eqⁿ.

$a_n - 5a_{n-1} + 6a_{n-2} = 1$. —(1)

As R.H.S is a constant, the particular solⁿ will be.

$a_n^{(P)} = A$. —(2)

$a_{n-1}^{(P)} = A$.

$a_{n-2}^{(P)} = A$.

Substituting in eqⁿ (1),

$A - 5A + 6A = 1$.

$+2A = 1$

$A = +1/2$

Substituting in eqⁿ (2),

$a_n^{(P)} = 1/2$.

$$\frac{16A + 20A + 6A}{16} = 42.$$

$$42A = 42 \times 16.$$

$$\boxed{A = 16.}$$

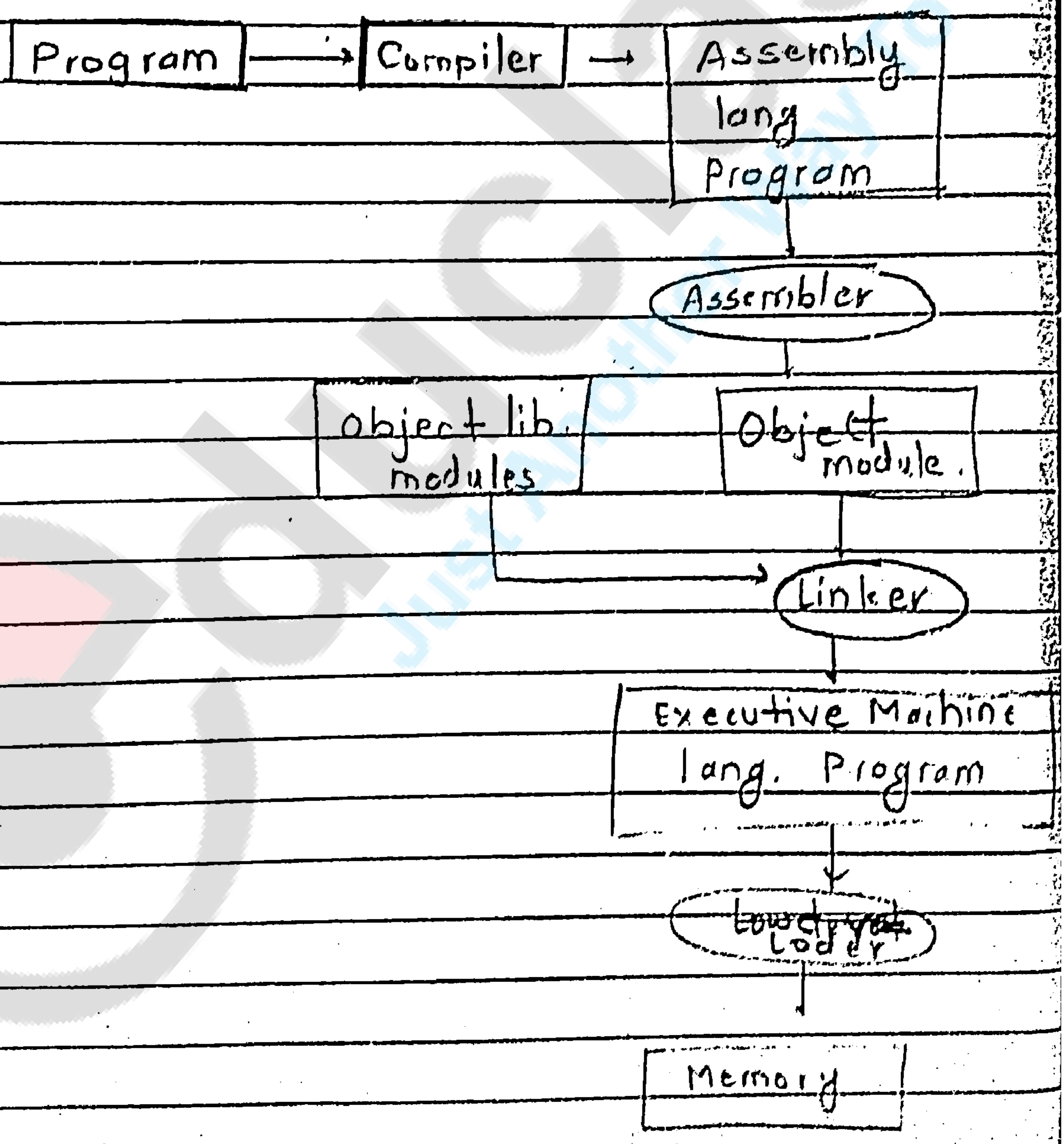
$$a_n^{(P)} = A \times 4^n$$

$$a_n^{(P)} = 16 \times 4^n.$$

I) Introduction to System s/w & OS.

Centralised OS.

Program execution Flow.



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Special case of PS.

if $f(n)$ is of the form
$(B_0 n^r + B_1 n^{r-1} \dots) m^n$.

where m is characterstics root of multiplicity k then

P.S is

$$n^k (A_0 n^r + A_1 n^{r-1} \dots + A_r) m^n$$

Eg. Find PS of $a_n - 2a_{n-1} = 3 \times 2^n$.

$(m-2)=0$

$m=2$ root of the ~~the~~ given eq^n ~~is~~ it is bcoz it is base of the exponential term on.

$\therefore$ RHS : $a_n = [An]2^n$

$\therefore$ F $\quad a_n - 2a_{n-1} = 3 \times 2^n \quad$ — (1)

$\therefore$ From eq^n 1 we get.

$$a_{n-1} = (A(n-1))2^{n-1}$$

eq^n (I) become

$$[An \cdot 2^n] - 2[A(n-1) 2^{n-1}] = 3 \times 2^n$$

$$An \cdot 2^n - A(n-1) 2^n = 3 \times 2^n$$

$$An - A(n-1) = 3$$

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(2) Find P.S of

$$a_n - 6a_{n-1} + 9a_{n-2} = (n+1)3^n \quad \text{—(1)}$$

$\rightarrow$

$$m^2 - 6m + 9 = 0$$
$$m^2 - 3m - 3m + 9 = 0$$
$$m(m-3) - 3(m-3) = 0$$
$$m-3 = 0 \quad \text{OR} \quad m-3 = 0$$
$$\underline{m=3} \quad \text{OR} \quad \underline{m=3}$$

But 3 is the base of expo. term in R.H.S

$\therefore$ The PS will be a special case & given

$$a_n^{(P)} = n^2(An+B)\,3^n$$

$$a_{n-1} = (n-1)^2(A(n-1)+B)3^{n-1}$$

$\rightarrow$ $n^2(An+B)3^n - 6[(n-1)^2(A(n-1)+B]3^{n-1}$

$$+ 9[(n-2)^2(A(n-2)+B]\,3^{n-2} = (n+1)3^n.$$

$\rightarrow$ $\cancel{3^n}\left[n^2(An+B) - \frac{6}{3}[(n-1)^2(A(n-1)+B] + \right.$

$$\left.\frac{9}{3}[(n-2)^2(A(n-2)+B]\right] = (n+1)\cancel{3^n}$$

$A = \frac{1}{4}$

$B = 5/4$

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(3) Find the total solution for the recurrence relation.

$$b_n = 4b_{n-1} + 1$$

with initial condition, $b_1 = 4$.

→

$$b_n = 4b_{n-1} + 1$$

$$b_n - 4b_{n-1} = 1 \quad \text{--- (1)}$$

char. eq^n is $m - 4 = 0$

$$\underline{m = 4}$$

Homogeneous sol^n is

$$a_n = A(4)^n$$

$$a_{n-1} = (A)(4)^{n-1}$$

R.H.S is constant, then P.S

$$b_n^{(P)} = B$$

$$b_{n-1} = B.$$

Substituting in eq^n (I), we get.

$$B = 4 \cdot B + 1$$

From eq^n (I),

$$B - 4B = 1$$

$$-3B = 1$$

$$B = -1/3 \qquad b_n^{(P)} = -1/3 \quad (4)$$

V $\therefore$ Total solⁿ is

$$T.S = H.S + P.S$$
$$= A4^n - \frac{1}{3}$$

$$b_n = A(4)^n - 1/3 \quad \text{--- (5)}$$

Initial condition, $b_1 = 4$.

$$b_1 = A(4)^1 - 1/3$$

$$4A - \frac{1}{3} = 4$$

$$4A = 4 + \frac{1}{3}$$

$$4A = \frac{13}{3}$$

$$A = \frac{13}{3} \times \frac{1}{4} = \frac{13}{12}$$

$\therefore$ Total solⁿ is

$$\boxed{b_n = \frac{13}{12}(4)^n - \frac{1}{3}}$$

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① Analytical Hierarchy Problem. (Tom Saaty)

Weight chart

Learning

Friends

Vocational training

College prep

Music class

	Learning	Friends	Sch. Life	Voc. train.	Colleg. prep	Music classes
L	1.00	4.00	3.00	1.00	3.00	4.00
F	0.25	1.00	1.00	0.33	1.00	2.00
S.	0.33	1.00	1.00	0.20	0.50	2.00
V	~~0.33~~ 1.00	3.00	5.00	1.00	1.00	3.00
C	~~1.00~~ ~~0.25~~ 0.33	3.00	2.00	1.00	1.00	2.00
M	0.25	0.50	0.50	0.33	0.50	1.00
	3.17	10.50	12.50	3.87	7.00	14.00

$$\frac{\left(\frac{1}{3.17}\right)+\left(\frac{4}{10.5}\right)+\left(\frac{3}{12.5}\right)+\left(\frac{1}{3.87}\right)+\left(\frac{3}{7}\right)+\left(\frac{4}{14}\right)}{6.}$$

	Learning	Friends	Sch. life	Voc. training	Coll. prep.	Music classes	Room Avg. (weg
L	0·32	0·38	0·24	0·28	~~0·29~~ 0·43	0·29	0·32.
F							
Sch. life							
Voc. trai							
Clg. prep							
Music classes							

Date: 13/09/2018

(2)

VIKOR method (Compromise Ranking Method)

A decision is to be made for choosing the best car among 4 cars. The following table represents the 4 alternatives car1, car2, car3 and car4 & 4 Attributes viz. Maintenance cost, Price in Durability, Resale Value. Use Vikor method to find best car.

Weights	1/2	1	1	1/2
	Maintenance cost in Rs.	Price in Rs.	Durability in years.	Resale value in Rs.
Car 1	1200	35000	5.5	100000
Car 2	1200	1000000	10	450000
Car 3	1250	6500	10	290000
Car 4	1850	500000	15	150000

Step 1 Check whether the weight are normalize.

Sum of weights $= \frac{1}{2} + 1 + \frac{1}{2} + 1 = 3$.

The sum of weight = 3. we need to normalize weight.

divide every weight of sum of weight.

After normalize we get the weights as

$\frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}$.

$\frac{1}{2}/3 = \frac{1}{6} = 0.333$ $\quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{2}/3 = \frac{1}{6} = 0.1666$

Step 2 : Identify the beneficiary and Non beneficiary attributes.

Car durability & Resale value are beneficiary (+)

Maintenance cost, Price are Non beneficiary (−)

If all are not beneficiary.

	Maintence cost in Rs	Price IN Rs	Durability in years	Resale Value in Rs.
Normalized weights	1/6	1/3	1/3	1/6
Cri type	−	−	+	+
Car 1	1200	350000	6.5	100000
Car 2	1000	1000000	10	450000
Car 3	1250	650000	10	290000
Car 4	1850	500000	15	150000
Mij) Max	1850	1000000	15	450000
Mij) Min	1000	350000	6.5	100000
Difference	850	650000	8.5	350000

~~Non-benificial~~ = (Max-current) / Difference.
Non-benificial = (Current - Min) / Difference

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For beneficiary attributes $= \dfrac{[(M_{ij})_{Max} - M_{ij}]}{\text{Difference}}$

For Non-beneficial attributes $= \dfrac{[(M_{ij}) - (M_{ij})_{Min}]}{\text{Difference}}$

For non-beneficial

$$\text{Car 1} = \frac{1200 - 1000}{850} = 0.235.$$

For beneficial, $15 - 6.5 / 8.5 = 1$

After Normalize

	(-)	(-)		(+)	(+)
Car 1	0.235	0	~~2.529~~	1	1
car 2	0	1	~~2.941~~	0.588	0
car 3	0.294	0.461	~~2.941~~	0.588	0.457
car 4	1	0.23	~~3.529~~	0	0.857.

Step3 We calculate E_i & F_i —R_i for each row.

$$E_1 = W1M11 + W2M12 + \cdots$$

$$R1 \text{ OR } F1 = \text{Max}(W1M11 + W2M12 + \cdots)$$

Intermediate matrix

Multiply element in each colum of decision matrix with weight.

Find E_i = Sum of the row.

F_i = Max of each row.

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	Maintenance cost in Rs.	Price in Rs.	Dur.	Resale Value	Row sum (E_i)	Row max. F_i or R_i
Car 1	0.039	0	0.333	0.167	0.539	0.333
Car 2	0	0.033	0.196	0	0.529	0.333
Car 3	0.049	0.154	0.196	0.076	0.475	0.196
Car 4	0.167	0.077	0	0.143	0.387	0.167

Calculate : Emax, E min, Fmax, Fmin.

E_i max = 0.539

E_i Min = 0.387

F_i Max = 0.333

F_i Min = 0.167

Step 4 Find the performance for each alternatives.

$$P_i = V\left[\frac{E_i - E_{min}}{E_{max} - E_{min}}\right] + (1-V)\left[\frac{F_i - F_{min}}{F_{max} - F_{min}}\right]$$

$E_i max - E_i min = 0.159$, $F_{max} - F_{min} = 0.166$

V = Vikor strategy value. (should be betn 0-1)

if value is not given assume, V = 0.5

$$P_i = 0.5\left[\frac{E_i - 0.387}{0.159}\right] + 0.5\left[\frac{F_i - 0.167}{0.166}\right]$$

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$$P_1 = 0.5\left[\frac{0.539 - 0.387}{0.159}\right] + 0.5\left[\frac{0.333 - 0.167}{0.166}\right]$$

$$= 0.5 \times \overset{0.152}{\cancel{0.955}} + 0.5 \times 1$$

$$= \cancel{0.076}$$

$$= 1$$

$P2 = 0.966$

$P3 = 0.376$

$P4 = 0.$

We choose ~~max~~ min of all Pi which is 0 for P4.

∴ P4 is minimum so the car 4 is the best alternative.

Step 5 Arronge the Ei, Fi & P in asending value.

③

ⅡⅠ SAW [Simple Additive Weighting]

Use SAW method to Find best fighter plane

Eg. ①

Attributes

C1 : maximum speed (Mach)

C2 : ferry range (NM)
i.e. maximum range that an aircraft can fly.

C3 : maximum payload (pounds)
i.e. Maximum Load the aircraft can carry.

C4 : purchasing cost ($*10^6)

C5 : reliability (high-low)

C6 : Maneuverability (high-low) (means planned and regulated movements)

Note :- Accept C4 all are beneficial.

	C1	C2	C3	C4	C5	C6
	0.2	0.1	0.1	0.1	0.2	0.3
	+	+	+	−	+	+
A1	2.0	1500	20000	5.5	Avg.	V. high
A2	2.5	2700	18000	6.5	low	avg
A3	1.8	2000	21000	4.5	high	high
A4	2.2	1800	20000	5.0	avg.	avg.

Weights - 0.2, 0.1, 0.1, 0.1, 0.2, 0.3

Weights are already normalized since total is 1.

Range		
	Very low	0
	Low	3
	Avg.	5
	High	7
	Very High	9

Total weight = 1
Weights are already normalized.

Quantified Mij

	↑	↑	↑	↑	↑	↑
	C1	C2	C3	C4	C5	C6
A1	2.0	1500	20000	5.5	5	9
A2	2.5	2700	18000	6.5	3	5
	9.5					
A3	1.8	2100	21000	4.5	7	7
A4	2.2	1800	20000	5.0	9	5

As all criteria are non-beneficiary, we normalize attributes.

For benificiary attributes (+) = Current value / largest

For non-benificary (−) = Min value / current value.

For given attributes.

benificiary

$C1 = max = 2.5$

$C2 = max = 2700$

$C3 = max = 21000$

~~C4~~ $C5 = max = 7$

$C6 = max = 9$

Non-beneficiary,

$C4 = 4.5 = min.$

After normalize

weights	0.2	0.1	0.1	0.1	0.2	0.3
	C1	C2	C3	C4	C5	C6
A1	0.8	0.5556	0.9524	0.8182	0.7143	1
A2	1	1	0.8571	0.6923	0.4286	0.5556
A3	0.7200	0.7407	1.0000	1	1	0.7778
A4	0.88	0.6667	0.9524	0.9000	0.7143	0.5556.

~~A1 = 0.8~~

We calculate the Pi for each row, for every element of the row we multiply by the weight in that column & find the sum of row.

For eg.

$A1 = (0.8)(0.2) + (0.56)(0.1) + (0.95)(0.1) + (0.82)(0.1)$

$+ ----$

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Alternative	Performance
A1	0.83547
A2	0.70733
A3	0.85141
A4	0.737743

In SAW method we take the highest value of P_i which is 0.85141.

∴ The A3 is the best alternative.

∴ Aircraft 3 is selected.

H.W

(2) Use saw method to suggest the best cutting flu

C.F	WW	TF	GT	SR	R	Th	EP	S
1	0.035	34.5	847	1.76	0.335	0.5	0.59	0.59
2	0.027	36.8	834	1.68	0.335	0.665	0.665	0.665
3	0.037	38.6	808	2.4	0.59	0.59	0.41	0.5
4	0.028	32.6	821	1.59	0.5	0.59	0.59	0.41
weights	60	13	30	15	10	25	25	10

DATE 15/03/2019

(4)

WSM [Weighted Sum Model]

Find the best alternative by using WSM model.

	C1	C2	C3	C4
Weight	0.20	0.15	0.40	0.25
A1	25	20	15	30
A2	10	30	20	30
A3	30	10	30	10

$A_1^{\text{WSM-score}} = 25 * 0.25 + 20 * 0.15 + 15 * 0.40 + 30 * 0.25$

$= 21.50$

$A2 = 22.00$

$A3 = 22.00$

Best Alternatives is A2, A3.

(5) Weighting Product Model.

WPM : Multicriteria Decision Making.

Solve by using WPM pairwise method.

Weights	0.20	0.15	0.40	0.25.
	C1	C2	C3	C4
A1	25	20	15	30
A2	10	30	20	30
A3	30	10	30	10

Pairs : A1/A2 , A1/A3 A2/A3.

We find the performace of each of the pairs by using the weighted Product methods.

$$P(A1/A2) = (25/10)^{0.20} * (20/30)^{0.15} * (15/20)^{0.40} * (30/30)^{0.25}$$

$$= (2.5)^{0.20} * (0.66)^{0.15} * (0.75)^{0.40} * (1)^{0.25}$$

$$= 1.007 > 1$$

$$P(A1/A3) = (25/30)^{0.20} \times (20/10)^{0.15} \times (15/30)^{0.40} \times (30/10)^{0.25}$$

$$= \cancel{1.007}\ 1.067 > 1$$

$$P(A2/A3) = 1.059 > 1$$

∴ A1 > A2 A1 > A3 , A2 > A3 order : A1 > A2 > A3

A1 is best alternative as value are more than any other option

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We can also find perform by WPM direct method

$P(A1) = \pi (M_{ij})^{wt.}$ for A1.

$P(A1) = 25^{0.2} \times 20^{0.15} \times 15^{0.4} \times 30^{0.25}$

$= 1.90 \times 1.56 \times 2.95 \times 2.34$

$= \underline{20.62}$.

$P(A2) = 10^{0.2} \times 30^{0.15} \times 20^{0.40} \times 30^{0.25}$

$= \underline{20.47}$

$P(A3) = 30^{0.20} \times 10^{0.15} \times 30^{0.40} \times 10^{0.25}$

$= 1.97 \times 1.41 \times 3.89 \times 1.77$.

$= \cancel{19.125}$ 19.33

Best is A1 > A2 > A3.

∴ The ~~highest~~ highest performance is A1.

DATE 18/03/2019.

Weight calculating using shannon's theory

Step 1 :- Normalize the decision matrix

$$R_{ij} = \frac{x_n}{\sum_{i=1}^{m} x_n}$$

Step 2 :- Calculate entropy value for each attribute e_j (entropy is it is the amount of decision ~~#~~ info. associated with each attribute)

$$e_j = -k \sum_{i=1}^{N} R_{ij} \ln R_{ij}$$

Where $\ln$ is long ~~th~~ log to the base e &

$$k = \frac{1}{\ln(N)}$$

Step 3 Calculate Degree of Diversification for each attribute $d_j = 1 - e_j$

Step 4 Calculate Weight for each attribute $W_j = \frac{d_j}{\sum_{i=1}^{m} d_k}$.

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Find the weights for the Decision matrix given below:

	C1	C2	C3	C4
A1	25	20	15	30
A2	10	30	20	30
A3	30	10	30	10

Step 1 : We normalize the decision matrix

Row sum

A1	90
A2	90
A3	80

Every row ~~dir~~ value divide by row sum.

R_{ij} =

	C1	C2	C3	C4
A1	$\frac{25}{90} \rightarrow$ 0.2778	0.2222	0.1667	0.333
A2	$\frac{10}{90} \rightarrow$ 0.1111	0.3333	0.2222	0.333
A3	$\frac{30}{80} \rightarrow$ 0.375	0.125	0.375	0.125

Step 2 :- $N = 3$ = $\log N$ = $\ln(3) = 1.099$

$K = 1/LN(N) = 1/1.099 = 0.91$

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1st Find log of Rij and multiply log value with Rij

$R_{ij} * LN(R_{ij}) = \ln(0.2778) * 0.2778$

	C1	C2	C3	C4
A1	−0.3558	−0.3342	−0.2999	−0.366
A2	0.2441	−0.3862	−0.334	−0.366
A3	−0.3678	−0.2599	−0.368	−0.26
	−0.9677	−0.9603	−1.001	−0.992

Step 3 Multiply each value of the column sum by −k. to get the ej value.

~~$d_j = 1 - e_j$~~

~~d_1~~

~~$d_j =$~~

−0.9677

x −0.91 →

e1	e2	e3	e4
0.88	0.87	0.91	0.9

Step 3 Calculate the degree of diversification.

$d_j = 1 - e_j$

	d1	d2	d3	d4	Sum
d_j =	0.12	0.13	0.09	0.1	0.44

$1 - 0.88$

Calculate weights.

Step 4 :- $W_j = \frac{d_j}{\sum_{i=1}^{m} d_j} = \frac{d_j}{0.44}$

W1	W2	W3	W4
0.273	0.295	0.205	0.227

$\frac{0.12}{0.44}$

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In a screnning test for a diesease. The $+_i$ of the dieases in a population 0.5%. The test is highly accurated with 5% false +ve rate & 10% false −ve rate. A person takes the test & it comes positive: construct a decision tree & use Bayes's theorm to determine the probability that the has a dieses?

Define the events:

$D+$: The person has a diesease.
D^- : The person does not have a disease.
$T+$: The test is positive.
T^- : The test is negative.

We have to find $P(D+)$.

$$P(D^-) = 1 - P(D^+)$$

$$P(D+) = 0.5\% = 0.005 \Rightarrow P(D-) = 0.995$$

False positive means the teste is +ve. the person does not have diesease.

$$P(T^+/D^-) = 5\% = \frac{5}{100} = 0.05$$

$$P(T^-/D^+) = 10\% = 0.1$$

From the above we can get complementary prob. if $P(T^-/D^-) = \underline{1 - P(T+/D-)}$

$$= 1 - 0.05$$
$$= \underline{0.95}$$

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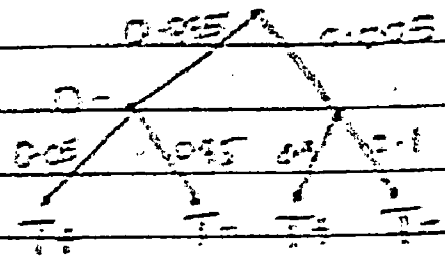

$P(D+) = 0.005$

$P(D-) = 0.995$

$P(T+|D-) = 0.05$

$P(T-|D-) = 0.95$

$P(T-|D+) = 0.1$

$P(T+|D+) = 0.9$

We have to find: $P(D+|T+)$

Using Bayes's theorm:

we have to find $P(D+|T+) = \dfrac{P(T+|D+) \cdot P(D+)}{P(T+)}$ —(1)

$P(T+) = (0.995 \times 0.05) + (0.005 \times 0.9)$

$= 0.05425$ —(2)

Substitute in eqn (1).

$$P(D+|T+) = \frac{P(T+|D+) \cdot P(D+)}{P(T+)}$$

$$= \frac{0.9 \times 0.005}{0.05425}$$

$$= 0.082947$$

$$= \underline{\underline{8.295\%}}$$

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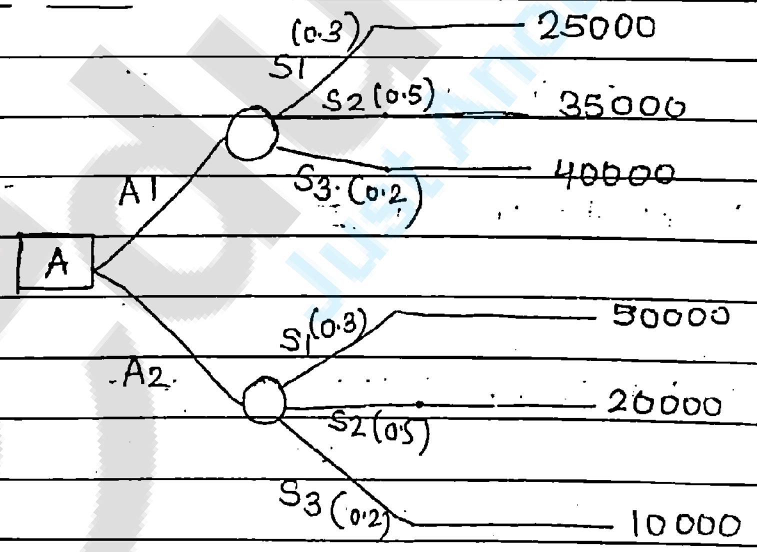
(1) Consider the following decision making problem
(1) Draw decision tree.
~~(2)~~ find Pay-off for every possible combination.

Pay-off Table.

Course of Action	State of Nature S1	S2	S3
A1	25000	35000	40000
A2	50000	20000	10000
Probability.	0.3	0.5	0.2

Decision Tree



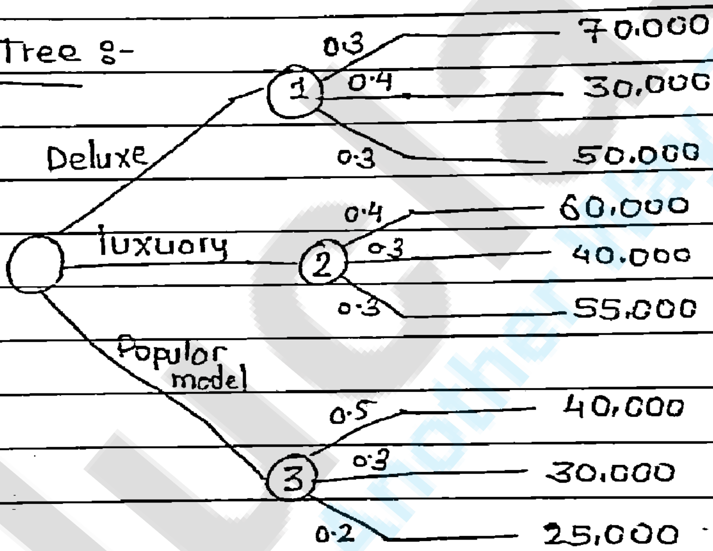
(2) Three models:

	Profit	Prob.
Deluxe	70k	0.3
luxuory	35k	0.4
toy.	50k.	0.3

	Profits			Probability.		
Deluxe	70k	30k	50k	0.3	0.4	0.3
luxuory	60k	40k	55k	0.4	0.3	0.3.
Popular model	40k	30k	25k	0.5	0.3	0.2

$\Rightarrow$

Decision Tree :-



EMV (Deluxe)
$= 70000 \times 0.3$
$+ 30{,}000 \times 0.4$
$+ 50{,}000 \times 0.3$
$50{,}000$

$$EMV(\text{Luxuory}) = 0.4 \times 60{,}000 + 0.3 \times 40{,}000 + 0.3 \times 55000$$
$$52{,}500$$

$$EMV(\text{Popular model}) = 0.5 \times 40000 + 0.3 \times 30{,}000 + 0.2 \times 25{,}000$$
$$34{,}000$$

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Rolling the Dice

① Suppose you are rolling a pair of dice once. If you roll a sum of 7 you win. If you roll anything else, you lose. It costs you $1 to play the game. If you win, you get your $1 back & an additional $5. Otherwise you lose $1.

If you make this bet 100 times over the course of the evening, how much do you expect to win? or lose?

→

There are two possible cases: win or lose.

We know that probability of event = Favourable outcome / total outcome

The possible sum we can get, can we shown in the following table.

Die 2 outcomes.

Die 1 outcomes	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

total no. of outcomes = 36.
Favourable outcomes of wins = 6.
[$\because$ 7 repeats 6 times]

Probability of wining $= \frac{\cancel{6}\,1}{\cancel{36}\,6}$

$= \frac{1}{6}$

Probability of loosing $= 1 - \frac{1}{6}$

$= \frac{5}{6}$

We need to calculate the payoff for wining & loosing.

Win → Payoff $x_1 = \$5 + \$1 - \$1 = \$5$

loose → Payoff $x_2 = -\$1$

	X	P(x)
winning	\$5	1/6
loosing	-\$1	5/6

$$E = \Sigma x \cdot P(x) = \left(5 \times \tfrac{1}{6}\right) + \left(-1 \times \tfrac{5}{6}\right) = 0$$

Whenever the expectation is zero. It means game is fair.

Suppose your charge is $2 to playgame, when you win the game you get back your $2 & additional $5 and when you loose you don't get anything

Payoff $X_1 = \$5 + \$2 - \$2 = \$5$

$X_2 = -\$2$.

	X	P(X)
wining	$5	1/6
loosing	-$2	5/6

$$E = \sum x \cdot P(x)$$

$$= \left(5 \times \frac{1}{6}\right) + \left(-2 \times \frac{5}{6}\right)$$

$$= \frac{-5}{6} = -0.83$$

loosing ~~strol~~ value = -0.83

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when you
additional
anything

- Decision theory, which is also called decision analysis, is a collection of mathematical models and tools developed to assist people in choosing among alternative actions in complex situations involving chance and risk.
- In the problem if situations & assumptions considered

Eg. 2 Roulette A common application of expected value is gambling.

American roulette wheel has 38 equally likely outcomes. The no.s are 0, 00, 1, 2, 3 --- 34, 35, 36. A winning bet placed on a single number pays 35 to 1. This means that you are paid 35 times your bet & your bet is returned so you get 36 times you bet after your bet is collected. So considering all 38 possible outcomes, find the expected value resulting from a \$1 bet on a single number.

→

If a ~~po~~ player bets on a single no in the American roulette game, there is a probability of 1/38 that the player wins.

If ~~he~~ he wins, he wins \$36 minus the \$1 costs of the bet.

He loses with probability 37/38, and he loses \$1.

Expected value,

$$E = (-\$1) * (37/38) + (\$36 - \$1) * (1/38)$$

$$= -0.0526$$

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Therefore one expects on avera-je to lose over five. For every dollar bet & the expected value of a one dollar bet is ($1 - $0.0526) or $.94.74c

In this case 1. since the expected value (-$0.0526) is -ve. the

25|03|2019

Stoping distance

The body is moving certain velocity & suddenly breaks are apply then the body stops completely after its cover a certing distance this is called as stoping distance.

The stopping distance depend upon road surface and reflexes of the cars driver & it is denoted by d.

It is actually the distance travelled betⁿ the time when body decided to stop moving the vehicle & the time when the vehicle actually stops completely.

Characteristic

$d \propto v^2$

① Stopping distance is directly proportional to v^2. V is speed or velocity of vehicle

② Inversly proportional to the friction betⁿ the surface of the tyres of the vehicle and the surface of the road.

i.e. if μ is coefficent of ~~fra~~ friction then $d = \frac{1}{\mu}$.

③ It is inversely proportional to gravitational acceleration g.

$$d \propto \frac{1}{g}.$$

combining three we get the formula for stopping distance as

$$\boxed{d = \frac{v^2}{2\mu g}}$$

~~Sometimes~~ μ & g are constant we also write as

$$\boxed{d = kv^2}$$

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Q.1 Car is moving with the velocity of 40 m/sec and breaks are suddenly apply. Determine the constant of proportanality if the body covers distance of 10 metrs before coming to rest.

$$d = kv^2$$

$$10 = k \times (40)^2$$

$$k = \frac{10}{1600} = \frac{1}{160}$$

Q.2 A bike moves with the velocity 15 m/sec and applies a break. Find the stopping distance if the constant of propornality 0.9.

$\boxed{g = 9.8}$ $S = \frac{d}{t}$

Q.3 A car is travelling at 100 km/hr when the driver sees something that requires a repeaid break. Find the total distance ~~if in the constant of proportionality~~ meters travelled by the car from the movement the driver 1st sees the prob. until the car stops. g = 10

Given intial speed $V = 100$ km/hr.

we converted to m/sec:

$$V = \frac{100 \times \overset{1km\ to\ m}{(10000)}}{\underset{hr\ to\ sec}{(60 \times 60)}} \text{ m/s}$$

$$= \frac{250}{9} \text{ m/s}$$

Drivers thinking time is 0.68 sec.

∴ thinking distance = Speed × tinking time

$$= \frac{250}{9} \times 0.68$$

$$= \underline{18.9\ m}$$

∴ breaking distance $= \frac{V^2}{2g} = \frac{(250/9)^2}{2 \times 10}$

$$= \underline{38.6\ m}$$

Stopping distance = thinking distance + breaking distance

$$= 18.9 + 38.6$$

$$= \underline{57.5\ m}$$

MCDM	MADM
① Refers to Multi Criteria Decision Making	Refers to Multi Attribute Decision Making
② Makes decision based on multiple usually conflicting criteria.	Makes decision based limited no. of pre-determined alternatives
③ Uses several criteria to rank objects.	MADM concerns objects that can be described through several attributes
④ MCDM method specifies how criteria could be ranked in order to arrive at a choice.	MADM specifies how attribute info. is to be processed in order to arrive at a choice
⑤ Used for making decision in quantitative as well as qualitative problem.	Used for making decision in quala qualitative problem.
⑥ MCDM involves MODM & MADM based on problem whether it is selection prob. or design prob.	MADM involves SAW, WPM & AHP.
⑦ Eg. When buying a car we could take criteria such as price horsepower	Eg. When buying a car, we could take criteria of accelation & attribute like car weight, horsepower & weight of car.

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AHP (Analytical Hierarchy Process)

DATE 26/03/2019

The board of directors have to choose a leader for a company whose founder is about to retire. There are three competing candidates TOM, DICK and HARRY and four competing criteria Experience Education, Charisma and Age. Use AHP to choose the most suitable candidate.

(The CMI, consistency index & consistency ratio need ~~to~~ not be calculated)

The comparision matrix for pair wise criteria is given below.

(if table is not given: take reciprocal (diagonal))

Criteria	Experience	Education	Charisma	Age
Experience	1	4	3	7
Education	1/4	1	1/3	3
Charisma	1/3	3	1	5
Age	1/7	1/3	1/5	1

Also the relative criteria for alternative is

Experience	Tom	Dick	Harry
Tom	1.00	1/4	4.00
Dick	4.00	1.00	9.00
Harry	1/4	1/9	1.00

Education	tom	Dick	harry
Tom	1.00	3	1/5
Dick	1/3	1.00	1/7
harry	5	7	1.00

Charisma	Tom	Dick	Harry
Tom	1.00	5	9.00
Dick	1/5	1.00	4.00
Harry	1/9	1/4	1.00

Age	Tom	Dick	Harry
Tom	1.00	1/3	5.00
Dick	3.00	1.00	9.00
Harry	1/3	1/9	1.00

$\Rightarrow$

Steps to Find Eigen vector or priority vector or weights.

1. Find column sum.
2. Find intermediate table : Element / columnsum
3. Weights = Avg. of Rows.

1. 1.72   8.33   4.53   16

Criteria Vs Criteria

Pairwise criteria table.

Experience	Education	Charisma	Age
1.72	8.53	4.53	16

Row Sum

① $\dfrac{\left(\frac{1}{1.72} + \frac{4}{8.53} + \frac{3}{4.53} + \frac{7}{15}\right)}{4} = \frac{2.1613}{4} = \frac{4}{}$

$= 0.5403$

② $\dfrac{\left(\frac{1}{1/1.72} + \frac{1}{8.53} + \frac{1/3}{4.53} + \frac{3}{15}\right)}{4} = 0.131.$

③ $= 0.272$

④ $= 0.058$

~~Thus~~ Experience	Edu.	Charisma	Age	Avg (ele/colum)
				0.54
				0.131
				0.272
				0.058
1.72	8.53	4.53	16	

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W/S : 0.54 0.131 0.272 0.058.

~~Critera~~

Alternative Vs Alternative.

Age	Tom	Dick	Harry	Relative Wt
Tom	1.00	1/3	5.00	0.267
Dick	3.00	1.00	9.00	0.669
Harry	1/5	1/9	1.00	0.064
column sum.	4.2	1.44	15	

Experience	Tom	Dick	Harry	Relative wt
Tom	1.00	0.25	4.00	
Dick	4.00	1.00	9.00	
Harry	0.25	0.11	1.00	
Col. sum	5.25	1.36	14	

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Education	Tom	Dick	Harry	Relative wt
Tom				
Dick				
Harry				
column sum				

Charisma	Tom	Dick	Harry	Relative wt.
Tom				
Dick				
Harry				
column sum				

Composite Impact table.

	Experience	Education	Charisma	Age
weights	0.54	0.132	0.272	0.058
Tom	0.22	0.193	0.735	0.267
Dick	0.713	0.083	0.2	0.669
Harry	0.067	0.723	0.065	0.064

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wt.

Each Element is multipled the column Weight and then the sum is taken.

1^{st} row.

$(0.22 \times 0.54) + (0.193 \times 0.132) + (0.735 \times 0.22\ldots$
$+ (0.267 \times 0.058)$

$= \underline{\underline{0.359}}$

wt.

Composite impact of	Tom	:	0.359
〃	Dick	:	0.489
〃	Horry	:	0.153

∴ Best composite score is:
Best Alternative is to choose : Dick.

28/03/2019

If you have to find consistency index or ratio. We have to follow sum additional step.

	0.54 Exp	0.131 Edu.	0.272 Char.	0.057 Age	Sum (wt * E/C)
Exp.	1	4	3	7	2.279
Edu	0.25	1	0.33	3	0.528
Char.	0.33	3	1	5	1.13
Age.	0.14	0.33	0.2	1	0.232
	1.72	8.33	4.53	16	

4/*E

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Col.6 Sum wt * Elem. ~~(elem / col.sum)~~ Row	col.7 ~~Avg.~~ ~~(Elem / col.sum)~~ wt	CMI. $\left(\frac{\text{col.6}}{\text{col.7}}\right)$ / $\frac{\text{sum}}{\text{wt}}$
2.279	0.54	4.2204
0.5277	0.131	4.0282
1.13	0.272	4.1544
0.2323	0.057.	4.0754

After calculating CMI we take the avg. of that which is called as λ_{max}.

Q. Find the consistency index for the following:

The comparison matrix for pair wise criteria is given below:

Criteria	Experience	Edu.	Chari	Age
Expr.	1	4	3	7
Edu.	1/4	1	1/3	3
Chari	1/3	3	1	5
Age	1/7	1/3	1/5	1

Step 1 : To Find the weight.

Find columnsum

	Expr.	Edu.	Char.	Age
Expr.	1	4	3	7
Edu.	0.25	1	0.33	3
Char.	0.33	3	1	5
Age	0.14	0.33	0.2	1
	1.7262	8.33	4.5333	16.

Steps for CI.

We write the weights horizontally also above the original table

① In org. table

Sum of CMI = 16.475

λ_{max} = The avg. of CMI = 16.475/4

$= \underline{\underline{4.11875}}$

Consistency Index = CI = $\dfrac{(\lambda max - \text{No. of rows})}{(\text{No. of rows} - 1)}$

$= \dfrac{4.11875 - 4}{3}$

$= \underline{\underline{0.039}} < 0.1$ (threshold value)

Note: If CI is zero then it means perfect consistency.

Find consistency index for Experience.

Exp:	Tom	Dick	harry	Sum	weights.

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Education — Weights

0.193

0.083

0.723

CI = 0.0327

Charisma

0.735

0.2

0.065.

CI = 0.0362

Age	Tom	Dick	harry	Sum (ele/col.sum)	wt.
	1	0.3333	.5	0.802 0.81	0.267
	3	1	9	2.006 2.046	0.669
	0.2	~~0.0~~ 0.1111	1	0.192 0.1927	0.064
	4.2	1.4444	15		

~~Sum of~~ CMI.

3.004

2.998

3

$\frac{}{9.002}$

Sum of CMI = 9.002.

$\lambda_{max} = \frac{9.002}{3}$

= 3.001

Consistency Index = 0.014

Recurrence Relation

Fast multiplication [by divide & ~~conqu~~ concur]

(1) 212×346

$(a+b)(c+d)$
$ac + ad + bc + bd$

$= (\underbrace{21 \times 10}_{a} + \underbrace{2}_{b}) \times (\underbrace{34 \times 10}_{c} + \underbrace{6}_{d})$

$= (21 \times 10 \times 34 \times 10) + (21 \times 10 \times 6) + (2 \times 34 \times 10) + (2 \times 6)$

$= (21 \times 34 \times 10^2) + 21 \times 10 \times 6 + 2 \times 34 \times 10 + 2 \times 6$

$= 714 \times 10^2 + (126 + 68)\,10 + 12$

$= 71400 + 1940 + 12$

$= \underline{73352}$

15 34500

(2) 2325×660

$= (\underbrace{23 \times 10^2}_{a} + \underbrace{25}_{b}) \times (\underbrace{6 \times 10^2}_{c} + \underbrace{60}_{d})$

$= (23 \times 10^2 \times 6 \times 10^2) + 23 \times 60 \times 10^2 + 25 \times 6 \times 10^2 + 25 \times 60$

$= (138)\,10^4 + 1380 \times 10^2 + 150 \times 10^2 + 1500$

$= \cancel{138}\ \underline{1,534,500}$

DATE 29/03/2019

Show that using rules of Inference show that it is tautology implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

$(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$
i ii iii

1. $P \vee Q$ — premise i
2. $\sim P \to Q$ — Implication.
3. $Q \to S$ — premise ii
4. $\sim P \to S$ (ii)(iii) & Hypothetical syllogism.
5. $\sim S \to P$ Contrapositive
6. $P \to R$ premise (ii)
7. $\sim S \to R$ 5, 6, H.S.
8. $\sim(\sim S) \vee R$
9. $S \vee R$

Q. Check validity of the following argument.
- If RAM has completed MCA or MBA then he is assured a good job.
- If RAM is assured a good job he is happy.
- RAM is not happy so RAM has not completed MCA.

→

let P denote RAM has completed MCA.
Q denote RAM has completed MBA.
R denote RAM is assured a good job.
S denote RAM is happy.

$P \vee Q \rightarrow R$ Premise(i)

$R \rightarrow S$ premise(ii)

$\sim S$

Prove $\sim P$

→

(I) $P \vee Q \rightarrow R$ (i)

(II) $R \rightarrow S$ (ii)

(III) $P \vee Q \rightarrow S$ H.S (i) (ii)

(4) $\sim S$ (iii)

⑤ $\sim(P \vee Q)$ 3, 4 & M.T

⑥ $\sim P \wedge \sim Q$

⑦ $\sim P$

- If I play football, I cannot study.
- Either i study or I pass statistics & Prob.
- I play football

Therefore I passed S&P.

→

P = I play football.

Q : I study.

R : I pass S & P.

DATE 01/04/2019

① Find the solⁿ of recurrence relation.

$a_n = 200a_{n-1} - 100$, with initial condition $a_0 = 1$.

$\Rightarrow$

Step 1 :- To find homogenous solⁿ.

$a_n - 200a_{n-1} = -100$ ① - standard form.

Homogeneous eqⁿ is,

$a_n - 200a_{n-1} = 0$.

$m - 200 = 0$.

$m = 200$

∴ The solⁿ of homogenous eqⁿ.

$A_n = A(m)^n$	$a_n = A$.
$A_n = A(200)^n$.	$a_n = An + B$.
	$a_n = An^2 + Bn + C$
	$a_n = A(200)^n$ — ②

R.H.S $= -100$.

Particular solⁿ.

For finding the particular solⁿ are s: so that the.

R.H.S $= -100$ which is const ~~that~~.

$(a_n)^p = B$.

$\therefore a_n - 1 = B$ — (3)

$\therefore$ eq (1) becomes.

$a_n = 200\, a_{n-1} - 100$

$a_n - 200 a_{n-1} = -100$ — (1)

eq^n (3) becomes, $a_n^{(p)} = \frac{100}{199}$ — (4)

T.S = H.S + P.S

$a_n = A(200)^n + \frac{100}{199}$ — (5).

put $n=0$ in eq^n (5),

$n=0,\ a_0 = A(200)^0 + \frac{100}{199}$

~~$a_0 =$~~

$1 = A \times 200^0 + \frac{100}{199}$

$1 = A + \frac{100}{199}$

(2) $1 - \frac{100}{199} = A$

$\frac{99}{199} = A$

$\boxed{A = \frac{99}{199}}$

sub. value of A in eq^n (5),

$a_n = \frac{99}{199}(200)^n + \frac{100}{199}$

$a_n = \frac{99(200)^n + 100}{199}$

(2) Solve the differential eq^n $a_n + 2a_{n-1} = n+3$.

where boundary $cond^n$ is $a_0 = 3$

$a_n + 2a_{n-1} = n+3$.

Standard form,

$a_n + 2a_{n-1} \cancel{-n} = \cancel{3} n+3$ — (1)

Homogeneous sol^n.

$a_n + 2a_{n-1} = 0$

$m + 2 = 0$

$m = -2$

We see that the R.H.S is linear eq^n.

$\therefore$ Homogeneous sol^n is,

$a_n = A(m)^n$

$a_n = A(-2)^n$ — (2)

→ To find P.S.

R.H.S : linear eq^n

$\therefore$ The P.S is.

$a_n^{(P)} = Bn + C$.

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$$a_{n-1}^{(P)} = B_{(n-1)} + B.$$

Substituting these value in eq^n (I).

$$(Bn + c) + 2[B(n-) + c] = n + 3.$$

$$Bn + C + 2Bn - 2B + 2C = n + 3.$$

$$3Bn - 2B + 3C = n + 3.$$

Equating same coefficient from R.H.S & L.H.S

$$3B = 1. \qquad \therefore B = 1/3.$$

$$-2B + 3C = 3$$

$$-2\left(\frac{1}{3}\right) + 3C = 3.$$

$$-\frac{2}{3} + 3C = 3.$$

$$3C = 3 + \frac{2}{3}$$

$$C = \frac{9+2}{3 \times 3} = \frac{11}{9}$$

$$\boxed{a_n^{(P)} = \frac{1}{3} n + \frac{11}{9}} \quad - (4)$$

add (2) & (4).

$$T.S = H.S + P.S$$

$$a_n = A(-2)^n + \frac{1}{3} n + \frac{11}{9} \quad - (5)$$

Initial condⁿ. $a_0 = 3$.., we put. $n=0$. in above (5) eqⁿ.

$$a_0 = 3 = A(-2)^0 + \frac{1}{3} \times 0 + \frac{11}{9}.$$

$$\cancel{3 = A(-2) +}$$

$$3 = A + \frac{11}{9}.$$

$$A = 3 - \frac{11}{9}.$$

$$= \frac{27-11}{9}$$

$$\boxed{A = \frac{16}{9}}$$

Sub. value A in eqⁿ (5), we get final T.S.

$$a_0 = \cancel{A}\,\frac{16}{9}(-2)^n + \frac{1}{3} \times n + \frac{11}{9}$$

Homogeneous

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① Find the homogeneous s^{ln} for recurrence relation

$$a_n = 2a_{n-1} - a_{n-2}$$

with initial condition $a_1 = 1.5$, $a_2 = 3$.

→

$a_n = 2a_{n-1} - a_{n-2}$

Step ① write eq^n in standard Form.

$a_n - 2a_{n-1} + a_{n-2} = 0$ —— ①.

Step ② Coefficient form.

$a_n - 2a_{n-1} + a_{n-2}$

$m^2 - 2m + 1 = 0$.

$(m-1)^2 = 0$.

$m = 1$ OR $m = +1$

Step ③ Real & equal, Say $m_1 = m_2 = m$.

$$a_n = (An + B)(m)^n$$

$$a_n = (An + B)(1)^n. \quad \text{—— II}.$$

If m repeated 3 times, $(An^2 + Bn + C)$

Initial $cond^n$,

① $a_1 = (A + B)(1)^1$

$1.5 = (A + B)$ —— ①

② $3 = (A2 + B)(1)^2$

$3 = 2A + B$ —— ②

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$$\cancel{A}+\cancel{B} = 1.5$$
$$2A + \cancel{B} = 3$$
$$\underline{-\quad -}$$
$$-A = -1.5$$
$$\boxed{A = 1.5}$$

$$A + B = 1.5$$
$$B = 1.5 - A$$
$$B = 1.5 - 1.5$$
$$\boxed{B = 0}$$

Sub. in eq^n II,

$$a_n = (An + B)(1)^n$$

$$a_n = 1.5n$$

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③ Find the solⁿ of the recurrence relation

$C_n + 6C_{n-1} + 12C_{n-2} + 8C_{n-3} = 0.$

Initial condⁿ, $C_0 = 1, C_1 = -2, C_2 = 8.$

→

$C_n + 6C_{n-1} + 12C_{n-2} + 8C_{n-3} = 0.$

1 6 12 8

$m^3 + 6m^2 + 12m + 8 = 0$ —①

Put, $m = -2.$

$(-2)^3 + 6(-2)^2 + 12(-2) + 8 = 0.$

$-8 + 24 - 24 + 8 = 0.$

$m = -2$

$m + 2 = 0.$

Synthetic divisi

-2	1	6	12	8
		-2	-8	-8
	1	4	4	0

$m^2 + 4m + 4 = 0.$

→ $(m+2)(m^2 + 4m + 4) = 0$

$(m+2)(m+2)^2 = 0.$

$m = -2, -2, -2$

m is repeated 3 times

$$a_n = (An^2 + Bn + C)(m)^n$$

$$a_n = (An^2 + Bn + C)(-2)^n \quad \text{— II}$$

$$C_n = (An^2 + Bn + C)(-2)^n \quad \text{— II}$$

Put $C_0 = 1$

$$C_0 = (A \cdot 0 + B \cdot 0 + C)(-2)^0$$

$$C = 1 \quad \text{— I}$$

$C_1 = -2$

$$\cancel{C_1 = (A(-2)^2 + B(-2) + C)(-2)^1}$$

$$\cancel{-2 = (4A - 2B + B)(-2)}$$

$$C_1 = (A(1)^2 + B(1) + C)(-2)^1$$

$$-2 = (A + B + C)(-2)$$

$$A + B + C = 1 \quad \text{— 2}$$

$C_2 = 8$

$$C_2 = (A(2)^2 + B(2) + C)(-2)^2$$

$$8 = (4A + 2B + C)(+4)$$

$$4A + 2B + C = 2 \quad \text{— III}$$

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From 1 & 2

$A+B=0$ $\times 2$. — ~~(4)~~

$4A+2B=1$

$4A+2B=1$

$2A+2B=0$ — 4

$2A+0=1$

$\boxed{A=\frac{1}{2}}$

put value of A in eqⁿ 4,

$\cancel{2A+2B=0}$

$\cancel{2\left(\frac{1}{2}\right)+2B=0}$

$\cancel{2B=0}$

$A+B+C=1$

$\frac{1}{2}+B+1=\phi$

$\boxed{B=\frac{-1}{2}}$

$A=\frac{1}{2}$, $B=\frac{-1}{2}$, $C=1$.

Real & distinct.

$$a_n=\left(\cancel{A}\left(\frac{1}{2}\right)n^2\cancel{B}-\frac{1}{2}n+C\right)(-2)^n.$$

$m_1=5$, $m_2=5$, $m_3=5$, $m_4=7$.

$$a_n=\left(An^2+Bn+C\right)(5)^n+D(7)^n$$

Particular Soln.

(3) Find the P.S for the recurrence relation

$$a_r + a_{r-1} = 3r \times 2^r \quad —(1)$$

Because R.H.S contain's exponential term we need to check if 2 is root of characteristic eqn.

homogeneous eqn is,

$$a_r + a_{r-1} = 0.$$

$$(m+1) = 0.$$

$$m = -1$$

As R.H.S is linear into exponential type.

$\therefore$ P.S is a_r

$$a_r^{(P)} = (Ar + B) \times 2^r \quad —(2)$$

$$a_{r-1} = (A_{r-1} + B) \times 2^{r-1}$$

divide by 2 & multiply by 2.

$$a_{r-1} = \frac{(A_{r-1} + B) \times 2^{r-1} \times 2}{2}$$

$$a_{r-1} = \frac{A_{r-1} + B \times 2^r}{2}$$

Put the value of a_r & a_{r-1} in eqⁿ (II),

$$\left[(Ar+B)\,2^r\right]+\left[\left(\frac{A(r-1)+B}{2}\right)\times 2^r\right]=3r\times 2^r.$$

$$\left[Ar+B\right]+\left[\frac{A(r-1)+B}{2}\right]=3r.$$

$$2Ar+2B+Ar-A+B=6r.$$

$$(3A)r+(3B-A)=6r.$$

$$(3A)r=6r.$$

$$3A=6$$

$$\boxed{A=2}$$

$$3B-A=0$$

$$3B-2=0.$$

$$\boxed{B=\frac{2}{3}}$$

$$a_r^{(P)}=\left(2r+\frac{2}{3}\right)\cdot 2^r.$$

Unit 2

1. Differential eqn.
   - H.S
   - P.S
   - T.S

2. Decision tree – AMP

3. MADM & MCDM. (methods – VIKOR, SAW, WSM) (last chapter) – 20

Quantifiers

The expression for all ($\forall$) & there exits ($\exists$) are called as quantifiers.

Predicate – A predicate is a stmt that contains one or more variable.

Proposition – If values are assign to the variables in a predicate then the resulting stmt is proposition.

For eg. $x < 5$ is predicate

$3 < 5$ is proposition.

Bound variable – A variable that has been quantified is set to be bound.

For eg.

There exist an x such that $x < 5$.

here x is variable which is bound by the quantifier 'there exist'

Sym. for :- $\exists x$, $P(x)$

Q. Write in symbol for the following english stmt.

(1) There exist an x such that $x < 4$.

$P(x)$: $x < 4$.

$\exists x \, P(x)$

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② For every number x, there is a number y such that $y = x + 1$

→

$P(x, y) : y = x + 1$

$\exists y, \forall x \; P(x, y)$

The quantifier for all is called universal quantifier and there exist is called as existential quantifier.

① 'Some rational numbers are real numbers.

$Q(x)$: x is rational number.
$R(x)$: x is real number

$\exists x \; Q(x) \rightarrow R(x)$

② let $A(x)$ denote x has white color.
$B(x)$: x is polar bear.
$C(x)$: x is found in cold.

i) There exist a polar bear whos color is not white.
ii) Every polar bear that is fond in cold region has white color.

→

i) $\exists x \; (B(x) \wedge \sim A(x))$

ii) $\forall x, \; [B(x) \wedge C(x)] \rightarrow A(x)$