Proposition

A proposition is a declarative sentence that is either true or false. It is also called as statement.

For eg.

P : Nashik is in Karnataka.
→ P is false.

Q : Belapur is in Navi-Mumbai
→ Q is true.

R : Bring me C++ book.
→ R is not an stmt.

P1 : Are you coming tomorrow ?
→ P1 is not a stmt / proposition.

// Connectives

We have the following connectives

1) Negation (NOT)
2) Conjunction (AND)
3) Disjunction (OR)
4) Conditional or implication (..if ..then)
5) Equivalence or biconditional (if and only if , iff)

1) Negation (NOT)

If P is a stmt. The negation of P is a stmt is NOT P.

It can be written in $\sim P$ OR $\neg P$

The truth table is given by.

P	$\sim P$
T	F
F	T

Binary

P	$\sim P$
1	0
0	1

2) Conjunction (AND) (Mul^n)

If P & Q are two statement. then their conjunction is statement of the form P & Q.

symbolic :- P AND Q , $P \wedge Q$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \wedge Q$
1	1	1
1	0	0
0	1	0
0	0	0

3) Disjunction (OR) (Add^n)

If P & Q are two stmt. then disjunction is statement of the form P OR Q & $P \vee Q$.

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

binary

P	Q	P ∨ Q
1	1	1
1	0	1
0	1	1
0	0	0

4) Conditional OR implication

IF P and Q are two stmt then their conditional stmt is stmt of the form 'if P then Q'.
symbblically :- $P \longrightarrow Q$

P	Q	$P \longrightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$P \longrightarrow Q$
1	1	1
1	0	0
0	1	1
0	0	1

5) Equivalance OR biconditional

IF P & Q are two stmt then their biconditional stmt is on the form is 'P iff Q' and P if & only if

Symbolic : $P \longleftrightarrow Q$.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

P	Q	① $P \to Q$	② $Q \to P$	1 ∧ 2.
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

① Find the TT for $P \to Q \to R$ where P, Q & R are proposition.

P	Q	R	$P \to Q$ ①	$① \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

HW ② $R \to P \to Q$ and $Q \to P \to R$

② Find the truth table for $P \leftrightarrow Q \leftrightarrow R$

P	Q	R	$P \leftrightarrow Q$ ①	$① \leftrightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	T
F	T	T	F	F
F	T	F	F	T
F	F	T	T	T
F	F	F	T	F

H.W ④ $Q \leftrightarrow P \leftrightarrow R$ and $R \leftrightarrow P \leftrightarrow Q$.

Antecedent and Consequent.

In $P \to Q$

P is Antecedent or hypothesis.

Q is consequent or conclusion.

$P \to Q$

also means

① if P then Q
② P implies Q
③ P only if Q
④ Q provided that P
⑤ Q if P
⑥ P is sufficient condition for Q.
⑦ Q is necessary condition for P.

If $P \to Q$ is conditional statement,

① Converse is

$Q \to P$

② Inverse is

$\sim P \to \sim Q$.

③ Contrapositive.

$\sim Q \to \sim P$

① Find the truth table for following, $P \to Q$ its converse, inverse and its contrapositive stmt.

P	Q	$P \to Q$	$Q \to P$ (converse)	$\sim P$	$\sim Q$	$\sim P \to \sim Q$ (Inverse)
T	T	T	T	F	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Contrapositive.

$\sim Q \to \sim P$

T
F
T
T

Eg. P : Today is sunday.
Q : It is a holiday.

Write $P \to Q$. Its converse, inverse and contrapositive.

→ $P \to Q$

If today is a sunday, then it is a holiday.

① Converse ($Q \to P$)

If it is a holiday then today is sunday.

$\sim P$: Today is not sunday.
$\sim Q$: It is not a holiday.

② Inverse. ($\sim P \to \sim Q$)

If today is not sunday then it is not holiday.

③ Contrapositive ($\sim Q \to \sim P$)

If it is not a holiday, then today is not sunday.

② If the proposition P denote ~~this book is good~~

P: This book is good.

Q: This book is costly.

Write the following stmt is symbolic form.

1) This book is good and costly.
2) This book is not good but costly
3) This book is cheap but good.
4) This book is neither good nor costly.
5) If this book is good then it is costly

Ans:-

① $P \wedge Q$

② $\sim P \wedge Q$

③ $\sim Q \wedge P$

④ $\sim P \wedge \sim Q$

⑤ $P \longrightarrow Q$

③ Find the truth value of the following proposition.

1) If 3 is not an integer, then $1/3$ is an integer.

2) If 3 is an integer, then $1/3$ is an integer.

3) 3 is an integer iff ~~o~~ $1/3$ is an integer

4) 3 is not an integer and $1/3$ is not an integer.

5) 3 is not an integer or $1/3$ is not an integer.

Ans:-

1) Truth value of 1st stmt is True.

2) False.

3) False.

4) False.

5) True.

H/W

① $R \rightarrow P \rightarrow Q$

P	Q	R	$R \rightarrow P$	$R \rightarrow P \rightarrow Q$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	F
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	T	F

② $Q \rightarrow P \rightarrow R$

P	Q	R	$Q \rightarrow P$	$Q \rightarrow P \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	F

HW

③ $Q \leftrightarrow P \leftrightarrow R$

P	Q	R	$Q \leftrightarrow P$	$Q \leftrightarrow P \leftrightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	T
F	T	T	F	F
F	T	F	F	T
F	F	T	T	T
F	F	F	T	F

④ $R \leftrightarrow P \leftrightarrow Q$

P	Q	R	$R \leftrightarrow P$	$R \leftrightarrow P \leftrightarrow Q$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	F
T	F	F	F	T
F	T	T	F	F
F	T	F	T	T
F	F	T	F	T
F	F	F	T	F

Laws of logic

① Equivalence law

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$$

② Implication law

$$P \rightarrow Q \equiv \sim P \vee Q$$

③ Double negation

$$\sim(\sim P) \equiv P$$

④ Idempotent law

(a) $P \wedge P \equiv P$

(b) $P \vee P \equiv P$

⑤ Commutative law,

(a) $P \wedge Q \equiv Q \wedge P$

(b) $P \vee Q \equiv Q \vee P$

⑥ Associative law,

(a) $(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

(b) $(P \vee Q) \vee R = P \vee (Q \vee R)$

⑦ Distributive law,

(a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

(b) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

⑧ Demorgan's law

(a) $\sim (P \wedge Q) = \sim P \vee \sim Q$

(b) $\sim(P \vee Q) = \sim P \wedge \sim Q$

⑨ Identity law,

a) $P \wedge T \equiv P$

b) $P \vee T = T$

⑩ Annihilation law

a) $P \wedge F = F$

b) $P \vee F = P$

⑪ Inverse law,

a) $P \wedge \sim P = F$

b) $P \vee \sim P = T$

⑫ Absorption law :

i) $P \wedge (P \vee Q) = P$

ii) $P \vee (P \wedge Q) = P$

P	Q	$P \wedge Q$	$P \vee (P \wedge Q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

⑬ Property of bicondition.

$P \leftrightarrow Q \equiv (P \vee Q) \vee (\sim P \wedge \sim Q)$

⑭ Contrapositive of a implication.

$P \rightarrow Q \equiv \sim Q \rightarrow \sim P$.

1) let P, Q and R be the following statemen

P : I drive over 65 miles per hour.
Q : I get a speeding ticket.
R : I am ~~agree~~ angry.

below are given english sentence Write them in symbolic form.

i) I get a speeding ticket only if I ~~dive~~ drive 65 miles per hr.

→ ∴ The above sentence also written as, If I drive 65 miles per hr then I get a speeding ticket.

If P then Q

∴ $P \rightarrow Q$.

ii) I do not drive over 65 miles per hr and I do not get speeding ticket yet I am angry.

→ $(\sim P \wedge \sim Q) \wedge R$

iii) I am angry, if I drive over 65 miles per hr and get a speeding ticket.

→ $(P \wedge Q) \rightarrow R$

Note :- whenever we take the negation,

1) All ⟷ Some.
i.e. for every ⟷ there exist

2) OR ⟷ And

$P \to Q \equiv \sim P \vee Q$.

P	Q	$P \to Q$	$\sim P$	$\sim P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

(2) $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$.

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$ (1)	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$ (2)
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

(3) $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$
T	T	T	F	F	F	F
T	F	F	F	F	T	F
F	T	F	F	T	F	F
F	F	F	T	T	T	T

Define functionally compelete set of connectives, give two examples.

There are five connectives:
$\sim, \vee, \wedge, \to, \leftrightarrow$

Some of these can be expressed in terms of a smaller set of connectives.

The set containing ~~mo~~ minimum number of connectives which are sufficient to express any given logical stmt in symbolic form is called as 'functionally complete set of connectives'

There two finctionally compelete set of connectives

i) $\{\sim, \vee\}$
ii) $\{\sim, \wedge\}$

(1) $\{\sim, \vee\}$

For eg.

(1) $\wedge$ can be expressed in terms of $\sim$ and $\vee$ and in the following way

$P \wedge Q \equiv \sim\sim(P \wedge Q)$ -- by double Negation law.

$\equiv \sim[\sim P \vee \sim Q]$ -- by demorgans law.

(2) Implication '$\longrightarrow$' can be expressed as,

$P \longrightarrow Q \equiv \sim P \vee Q$

(3) $\longleftrightarrow$ can be expressed as,

$P \longleftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$ -- equivalence law.

$\equiv \underbrace{(\sim P \vee Q)}_{A} \wedge \underbrace{(\sim Q \vee P)}_{B}$

$\equiv \sim(\sim(\sim P \vee Q)) \vee [\sim(\sim Q \vee P)]$ --

$- (A \wedge B) \equiv \sim[\sim A \vee \sim B]$

$\{\sim, \vee\}$ this set is functionally complete set of connectives.

(2) Prove $\{\sim, \wedge\}$ is a functionally complete set of connectives.

i) $\vee$

$P \vee Q \equiv \sim\sim(P \vee Q)$

$\equiv \sim[\sim P \wedge \sim Q]$ -- De-morgan's

ii) $\longrightarrow$

$P \longrightarrow Q \equiv \sim P \vee Q$

$\equiv \sim\sim[\sim P \vee Q]$

$\equiv \sim[\sim\sim P \wedge \sim Q]$

$\equiv \sim[P \wedge \sim Q]$ -- by eq^n (I)

iii) $\longleftrightarrow$

$P \leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$ -- Equivalence.

$\equiv [\sim(P \wedge \sim Q)] \wedge [\sim(Q \wedge \sim P)]$ -- from eq^n (II)

① Toutology

A toutology or a universaly true formula is a well formed formula. who's truth value is always true.

② contradiction

A contradiction or absurdity is well formed (WFF) formula. who's truth value is always false.

Que. Determine weather the following toutology or contradiction.

1) $\underbrace{(P \rightarrow Q)}_{①} \wedge \underbrace{(P \wedge \sim Q)}_{②}$ Using TT.

P	Q	$P \rightarrow Q$	$\sim Q$	$P \wedge \sim Q$	① $\wedge$ ②
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

It is a contradiction.

(ii) $(P \rightarrow Q) \wedge (P \wedge \sim Q)$ Using law

$\Rightarrow$

$\equiv (\sim P \vee Q) \wedge (P \wedge \sim Q)$ -- Implication

$\equiv (\sim P \vee Q) \wedge [\sim\sim(P \wedge \sim Q)]$ -- double neg.

$\equiv \underbrace{(\sim P \vee Q)}_{A} \wedge [\sim \underbrace{(\sim P \vee Q)}_{\sim A}]$ -- Demorgan's

$\equiv A \wedge \sim A.$ -- Inverse law.

$\equiv F.$

OR

$(P \rightarrow Q) \wedge (P \wedge \sim Q)$

$\equiv (\sim P \vee Q) \wedge (P \wedge \sim Q)$

$\equiv \underbrace{(P \wedge \sim Q) \wedge (\sim P \vee Q)}_{A \wedge (B \vee C)}$

$\equiv [(P \wedge \sim Q) \wedge \sim P] \vee [(P \wedge \sim Q) \wedge Q]$

$\equiv [P \wedge \sim P \wedge \sim Q] \vee [P \wedge \sim Q \wedge Q]$

$\equiv [F \wedge \sim Q] \vee [P \wedge F]$

$\equiv F \vee F$

$\equiv F.$

② Test weather a following is Tatutology, contradiction or contegency.

$[P \to (Q \to R)] \to [(P \to Q) \to (P \to R)]$

P	Q	R	① $Q \to R$	② $P \to (Q \to R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

③ $P \to Q$	④ $P \to R$	⑤ ③ → ④	② → ⑤
T	T	T	T
F	F	F	T
F	T	T	T
F	F	T	T
T	F	T	T
T	T	T	T
T	T	T	T
T	T	T	T

$\wedge$ Product → conjunction
$\vee$ sum → disjunction

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Min terms are fundamental conjunctions.

A min terms in n posi propositional variables.

$P_1, P_2, --- P_n$

$Q_1 \wedge Q_2, -- \wedge Q_n$

where each Q_i is either P_i or $\sim P_i$

Eg. $P, \sim P, \sim P \wedge Q, P \wedge Q, P \wedge \sim P \wedge Q,$
$P \wedge Q \wedge \sim R$ --

Literal → $P, \sim P, Q, \sim Q, R, \sim R$.

Max term are fundamental disjunction.

A max term is is in n propositional variable $P_1, P_2, --- P_n$

$Q_1 \vee Q_2$ ---- $\vee Q_n$.

where each Q_i is either P_i or $\sim P_2$

eg. $P \sim P$, $\sim P \vee Q$, $P \vee \sim P \vee Q$ -- etc.

Elementary Sum & Elementary Product.

An elementry product is a product of literals and elementary sum is sum of literals.

For two variables P & Q the elementary product are $P \wedge \sim Q$, $\sim P \wedge Q$, $\sim Q \wedge \sim P$, $P \wedge Q$

For two variable P & Q the elementary sums are $P \vee \sim Q$, $\sim P \vee Q$, $\sim Q \vee \sim P$, $P \vee Q$.

Disjunctive Normal Form : DNF [SOP]

~~If~~ a formula is in DNF if it is a sum of elementary product

Eg. i) $P \vee (Q \wedge R)$

ii) $P \vee (\sim Q \wedge R)$

iii) $(\sim P \wedge Q) \vee R$

Rules :-

1) To obtain a formula in DNF form, we need to eliminate implication or biconditio[nal]

2) Use demorgan's law to eliminate negation before the sums or products. The resulting formula has negation only before ~~only~~ proposition variable.

3) Apply distributed law repeatedly till you get a DNF.

Obtain DNF of the following.

1) $P \vee (\sim P \rightarrow (Q \vee (Q \rightarrow \sim R))$

→ We use implication law,

$\equiv P \vee (\sim P \rightarrow (Q \vee (\sim Q \vee \sim R))$

$\equiv P \vee (\sim P \rightarrow (\underbrace{Q \vee \sim Q}_{T} \vee \sim R))$

$\equiv \cancel{P \vee} (\cancel{\sim} P \cancel{\rightarrow} \cancel{T})$

$\equiv P \vee (P \vee (Q \vee \sim Q \vee \sim R))$

$\equiv \underline{\underline{P \vee P}} \vee Q \vee \sim Q \vee \sim R.$

$\equiv P \vee Q \vee \sim Q \vee \sim R.$

ii) $P \wedge (P \rightarrow Q)$

$\equiv P \wedge (\sim P \vee Q)$ -- Implication law

$\equiv (P \wedge \sim P) \vee (P \wedge Q)$ -- distributive law.

#

iii) $P \wedge \sim(Q \wedge R)) \vee (P \rightarrow Q)$

$\equiv P \wedge \sim(Q \wedge R)) \vee (\sim P \vee Q)$ -- Implication.

$\equiv P \wedge (\sim Q \vee \sim R)) \vee (\sim P \vee Q)$

$\equiv (P \wedge \sim Q) \vee (P \wedge \sim R) \vee (\sim P \vee Q)$ -- distributive

$\equiv (P \wedge \sim Q) \vee (P \wedge \sim R) \vee \sim P \vee Q\}$

iv) $\underbrace{\sim(P \vee Q)}_{P} \leftrightarrow \underbrace{(P \wedge Q)}_{Q}$ - $P \leftrightarrow Q \equiv (P \wedge Q) \vee (\sim P \wedge Q)$

$\equiv [\{\sim(P \vee Q) \wedge (P \wedge Q)] \vee [(P \vee Q) \wedge \sim(P \wedge Q)]$

$\equiv [(\sim P \wedge \sim Q) \wedge (P \wedge Q)]\ [(P \vee Q) \wedge (\sim P \vee \sim Q)]$

Prove that following tautology.

$\sim(P \leftrightarrow Q) \longleftrightarrow [(P \wedge \sim Q) \vee (Q \wedge \sim P)]$

If you have to prove the above stmt as tautology, as we have bicondn in tautology

It is sufficient if we take tautology use law and bring the RHS.

L.H.S : $\sim(P \leftrightarrow Q)$

$\equiv \sim[(P \rightarrow Q) \wedge (Q \rightarrow P)]$ -- Equivalonce

$\equiv \sim[(\sim P \vee Q) \wedge (\sim Q \vee P)]$ -- Implication

$\equiv \sim(\sim P \vee Q) \vee \sim(\sim Q \vee P)$ -- Demorgan's law.

$\equiv (P \wedge \sim Q) \vee (Q \wedge \sim P)$ -- by demorgan's and double negation

$\equiv$ R.H.S.

≑ Principal Disjunction Normal Form :

Obtain the principal DNF of α where $\alpha = P \vee (\sim P \wedge \sim Q \wedge R)$

Step:- 1) we First form the truth table of α

P	Q	R	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q \wedge R$ (1)	α = $P \vee$ (1)
T	T	T	F	F	F	T
T	T	F	F	F	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

2) Mark the ~~column~~ rows which are true.

Principal D.NF =

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R)$
$\vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$

Find the principal D.NF of α

$\alpha = (\sim P \rightarrow R) \wedge (P \leftrightarrow Q)$

$= \boxed{(\sim P \rightarrow R) \wedge [(P \wedge Q) \vee (\sim P \wedge \sim Q)]}$

P	Q	R	$\sim P$	$\sim Q$	$\sim P \rightarrow R$ (1)	$P \wedge Q$ (2)
T	T	T	F	F	T	T
T	T	F	F	F	T	T
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	T	F	T	F	F	F
F	F	T	T	T	T	F
F	F	F	T	T	F	F

$\sim P \wedge \sim Q$ (3)	(2) $\vee$ (3) (4)	(1) $\wedge$ (4)
F	T	T
F	T	T
F	F	F
F	F	F
F	F	F
F	F	F
T	T	T
T	T	F

$\therefore (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$

Principal Conjunction Normal Form.

The formula α is in principal C.N.F if α is product of max terms.
To obtained principal C.N.F we find principal D.N.F of $\sim\alpha$ and again apply negation.

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Que① Find Principal C.N.F of α.

$(P \not\to Q) \not\equiv (P \not\to Q)$

$$\alpha = (\sim P \vee \sim Q) \to (P \leftrightarrow Q)$$

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$P \leftrightarrow Q$	α	$\sim\alpha$
T	T	F	F	F	T	T	F
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	F

DNF of $\sim\alpha$.

$$\sim\alpha = (P \wedge \sim Q) \vee (\sim P \wedge Q)$$

Principal CNF of α
is $\sim(\sim\alpha) = \sim\boxed{[(P \wedge \sim Q) \vee (\sim P \wedge Q)]}$

$$\alpha = \sim(P \wedge \sim Q) \wedge \sim(\sim P \wedge Q)$$

$$\alpha \equiv (\sim P \vee Q) \wedge (P \vee \sim Q).$$

product of sums.

② Find principal CNF and DNF of α
where $\alpha = P \vee (Q \to R)$

P	Q	R	$Q \to R$	$P \vee (Q \to R)$ $\alpha =$	$\sim\alpha$
T	T	T	T	T	F
T	T	F	F	T	F
T	F	T	T	T	F
T	F	F	T	T	F
F	T	T	T	T	F
F	T	F	F	F	T
F	F	T	T	T	F
F	F	F	T	T	F

To Find a principal DNF of α,
consider the truth value of α.

$\alpha = (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R) \vee$
$(P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee \cancel{(\sim P \wedge Q \wedge \sim R)}$
$\vee (\sim P \wedge \sim Q \wedge R) \vee (\sim P \wedge \sim Q \wedge \sim R)$

For principal CNF we consider column for $\sim\alpha$.

① DNF of $\sim\alpha$.

$$\sim\alpha = (\sim P \wedge Q \wedge \sim R)$$

2) principal of α

$\sim(\sim\alpha) = \sim(\sim P \wedge Q \wedge \sim R)$

$\boxed{\alpha = (P \vee \sim Q \vee R)}$

3) Find the principal CNF and DNF of α where $\alpha = (\sim P \vee \sim Q) \rightarrow (P \leftrightarrow \sim Q)$

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$ (1)	$P \leftrightarrow \sim Q$ (2)	α	$\sim\alpha$
T	T	F	F	F	F	T	F
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	T	F	F	T

Principal DNF of α.

$\alpha = (P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

Principal CNF of α.

D.N.F of $\sim\alpha$.

$\sim\alpha = (\sim P \wedge \sim Q)$

principal CNF of α.

$\sim(\sim\alpha) = \sim(\sim P \wedge \sim Q)$

$= P \vee Q$

3) Prove that the following is tautology without using truthtable.

$(P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R)) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$

$\longrightarrow$

$\equiv (P \vee Q) \wedge [P \vee \sim(\sim Q \vee \sim R)] \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$

$\equiv (P \vee Q) \wedge [P \vee (Q \wedge R)] \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$

$\equiv (P \vee Q) \wedge [(P \vee Q) \wedge (P \vee R)] \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$

$\equiv [(P \vee Q) \wedge (P \vee R)] \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$

$\equiv [(P \vee Q) \wedge (P \vee R)] \vee \sim[\sim(\sim P \wedge \sim Q) \wedge \sim(\sim P \wedge \sim R)]$

$\equiv [(P \vee Q) \wedge (P \vee R)] \vee \sim[(P \vee Q) \wedge (P \vee R)]$

$\equiv T$ -- $(A \vee \sim A = T)$

4) If $(P \leftrightarrow Q) = T$ Find the truthvalue of $\sim(P \vee Q) \leftrightarrow \sim Q$

we know that $(P \leftrightarrow Q)$ is true

Case (I) 1) if P & Q both are true and

2) if P & Q both are false.

$\alpha = \sim(P \vee Q) \leftrightarrow \sim Q$

$\alpha = (\sim(P \vee Q) \wedge \sim Q) \vee ((P \vee Q) \wedge Q)$ ... bicondition

(1) $P = T$ & $Q = T$

$\alpha = (\sim(T \vee T) \wedge F] \vee [(T \vee T) \wedge T]$

$= (F \wedge F) \vee (T \wedge T)$

$= F \vee T$

$= \underline{T}$

(2) $P = F$ & $Q = F$

$\alpha = (\sim(F \vee F) \wedge T) \vee ((F \vee F) \wedge F))$

$= (T \wedge T) \vee (F \wedge F)$

$= T \vee F$

$= T$ //

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Mathematical Induction

$P(n)$ is a property which is true for all n then we can prove it by method of mathematical Induction by using the following steps

Steps (1) show $P(1)$ is true.
(2) Assume $P(k)$ is true for some k.
(3) Prove $P(k+1)$ is true
$\Rightarrow P(n)$ is true for all n.

~~Use Mathe~~

Use Mathematical induction

1) If $3n+2$ is odd number then n is odd where n is a natural no.

let $P(n)$ be the property, if $(3n+2)$ is odd then n is odd.

Step 1

$P(1) = 3(1)+2$
$= 3+2 = 5$ //

5 is odd then 1 is odd.

let $P(k)$

Assume $P(k)$ if $3k+2$ is odd then k is odd, is true. (1)

Step 3 To prove $P(k+1)$ is true.

$P(k+1)$: if

Claim $P(k+1)$ if $\{3(k+1)+2$ is odd then $k+1$ is odd

Given:-

$3(k+1)+2$ is odd

$= 3k+3+2$ is odd.

$= 3k+2+3$ is odd

→ but $(3k+2)$ is odd then k is odd

→ but the next odd no ~~is~~ after k is k+2.

→ so here we need to prove $3k+2+6$ is odd no. ($\because$ odd + even) = odd

which can be written as $3k+2+6$,

$3k+2+6 = 3(k+2)+2$ is odd

$3(k+2)+2 = 3n+2$

where $n = k+2$

Thus the proprety true for $n = k+2$

proved by mathematical induction.

① $P(n) : 1+2+3+4+\ldots+n = \frac{n(n+1)}{2}$

① L.H.S : 1

R.H.S : $\frac{1(1+1)}{2} = \frac{2}{2} = 1$.

② ~~P(k) =~~

Assume $P(k) = 1+2+\ldots+k = \frac{k(k+1)}{2}$ is true.

③ $P(k+1) = 1+2+\ldots+k+k+1 = \frac{(k+1)(k+1+1)}{2}$

L.H.S : $1+2+\ldots+k+k+1$

R.H.S : $k\frac{(k+1)}{2} + (k+1)$

$= (k+1)\left[\frac{k}{2}+1\right]$

$= \frac{(k+1)(k+2)}{2}$

$= \frac{(k+1)(k+1+1)}{2}$

= L.H.S

② Let $P(n) = 1+2+2^2+\ldots+2^n = 2^{n+1}-1$

→ Step ① let us check P(1) is true.

L.H.S : $P(1) : 1+2^1 = 3$.

R.H.S : $P(1) = 2^2 - 1 = 3$.

② $P(k) : 1+2^1+2^2+\ldots+2^k = 2^{k+1}-1$ is true.

T.P.T : P(k+1) is true.

T.P.T : $P(k+1) : 1+2^1+\ldots+2^k+2^{k+1} = 2^{k+1+1}-1$

L.H.S OF $P(k+1)$

L.H.S : $1 + 2^1 + \cdots + 2^k + 2^{k+1}$

$= (2^{k+1} - 1) + 2^{k+1}$

$= 2^{k+1} - 1 + 2^{k+1}$

$= 2 \cdot 2^{k+1} - 1$

$= 2^{(k+1)+1} - 1$

= R.H.S.

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Relation R

R from set A to set B is a subset of A×B.

e.g. $A = \{1,2,3,4\}$
$B = \{a,b\}$.

$A \times B = \{(1,a) (1,b), (2,a) (2,b), (3,a), (3,b), (4,a), (4,b)\}$

$R_1 = \{(2,a), (4,b), (3,a)\}$

$R_2 = \{(3,a), (3,b)\}$

if $(x,y) \in R$ then we write xRy it means x is Related to y under Relation R.

M_{R_1}	a	b
1	0	0
2	1	0
3	1	0
4	0	1

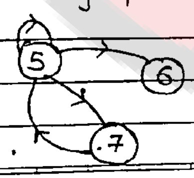
M_{R_2}	a	b
1	0	0
2	0	0
3	1	1
4	0	0

If both the sets a & b are same then we call it relation on A.

e.g. $A = \{5,6,7\}$

$R = \{(5,6), (5,7), (7,5), (5,5)\}$

we can represent such types of relation with diagraphs also.



Domain of Relation (R)

The domain of R is denoted by dom(R) and it is set of element $a \in A$ if $(a,b) \in R$

$A = \{1,2,3,4\}$ $B = \{a,b\}$

$R_1 = \{(2,a)\ (3,b)\ (4,a)\}$

$dom(R_1) = \{2,3,4\}$

Range of a Relation (R)

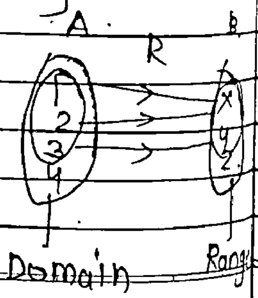
The range of R is denoted by Ran(R).

It is set of elements $b \in B$ if $(a,b) \in R$.

$A = \{1,2,3,4\}$ $B = \{x,y,z\}$

$R_1 = \{(1,x)\ (2,x)\ (3,y)\}$

$Ran(R_1) = \{x,y\}$



Types of Relation

1) Equality Relation

It is denoted by δ, a relation R on set a is an equality Relation. If its consist of only the ordered pairs (a,a) where $a \in A$. $\Delta : A \longrightarrow A$

eg. $A = \{7,8,9\}$

$A = \{(7,7),(8,8),(9,9)\}$

$$\text{Matrix} = \begin{array}{c} \\ 7 \\ 8 \\ 9 \end{array} \begin{array}{c} \begin{array}{ccc} 7 & 8 & 9 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

2) Reflective

A reflective relation on a set A is reflective if $(a,a) \in R$ for $a \in A$.

$A = \{7,8,9\}$

$R_1 = \{(7,7)(7,8)(8,8)(9,9)(8,9)\}$

$A = \{(7,7)(8,8)(9,9)\}$

σ is subset of reflective relation. Rather we can say the relation will reflective if & only if σ is its subset.

3) **IRReflective**

A Relation R on set A is IRReflective relation does not belong to R for $(a,a) \notin R$.

$A = \{a, b, c\}$

$R_2 = \{(a,b), (b,c), (a,c)\}$

4) Symmetric

Relation R on set A is symmetric if whenever $(a,b) \in R$ we must have $(b,a) \in R$.

5) Asymmetric Relation

A relation R on set A is asymmetric relation on if whenever $(a,b) \in R$, $(b,a) \notin R$

Note: In Asymmetric relation we can not have relation (a,a)

6) Antisymmetric relation.

A relation R on set A is called an Antisymmetric relation if whenever $(a,b) \in R$ & $(b,a) \in R$ we must have $a = b$.
It is same asymmetric where diagonal elements are allowed.

e.g. $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,2), (2,3), (3,2), (2,1)\}$

$R_2 = \{(1,1), (2,3), (4,4)\}$

$R_3 = \{(1,3), (3,2), (4,1)\}$

$R_4 = \{(1,1), (2,3), (3,2)\}$

R_1 is Symmetric, not asymmetric & antisymmetric.

R_2 is Antisymmetric

R_3 is Asymmetric and Antisymmetric.

R_4 is Antisymmetric

$R_5 = \{(1,1)(2,2)\}$ → Symmetric and Antisymmetric

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7) Inequality

let R be a relation on set A where $R = \{(a,a) \in A \times A \,/\, a \neq b\}$

$A = \{1,2,3\}$

$R = \{(1,2)(1,3)(2,3)(2,1)(3,1)(3,2)\}$

8) Empty set

let (a,b) non-empty set, then $R = \phi \in A \times A$ then R is an empty relation.

9) Transitive Relation

let R be a relation on set A. It is called as transitive relation if aRb & bRc then aRc

i.e. $(a,b) \in R$ & $(b,c) \in R$ them $(a,c) \in R$

e.g. ① $A = \{1,2,3\}$

$R = \{(1,1),(1,2),(2,3),(13)\}$

② $A = \{1,2,3,4\}$

$A = \{(1,1)(1,3)(2,3)(3,2)\}$

$(1,3) \in R$

$(3,2) \in R$

but $(1,2) \notin R$

1) Let R be the Relation on set A given by

$A = \{1,2,3,4\}$

$R = \{(1,1),(2,2),(1,2)(3,4),(4,4)\}$

Identify the types of relation and draw matrix and also draw digraphs.

→

① It is not an equality relation because $(3,3) \notin R$ & $(1,2)(3,4) \in R$

② It is not an inequality relation.

③ It is not an empty relation. $R \neq \phi$

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④ As $(3,3) \notin R$ the relation is not reflective.

⑤ because $(1,1)(2,2)(4,4) \notin R$
$\therefore$ It is not Irreflective relation

⑥ It is not symmetric relation.
since $(1,2) \in R$ but $(2,1) \notin R$

⑦ It is not asymmetric relation.

⑧ It is an antisymmectric relation.
$\because$ whenever $(a,b) \in R$ & $(b,a) \in R$
we have $a = b$

⑨ $\therefore$ The relation is transitive relation

$\overset{a\ b}{(1,1)} \in R$	$\overset{b\ c}{(1,2)} \in R$	$(1,2)$
$(2,2) \in R$		
$(1,2) \in R$	$(2,2) \in R$	$(1,2)$
$(3,4) \in R$	$(4,4) \in R$	$(3,4)$
$(4,4) \in R$	$(4,4) \in R$	$(4,4)$

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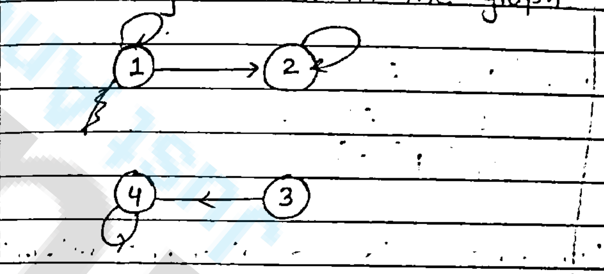
$$M_R = \begin{array}{c|cccc|c} & 1 & 2 & 3 & 4 & \text{Out deg.} \\ \hline 1 & 1 & 1 & 0 & 0 & 2 \\ 2 & 0 & 1 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 0 & 1 & 1 \\ \hline \text{Indeg} = & 1 & 2 & 0 & 2 & \end{array}$$

To find the Indegree & out degree for every vertex in the graph.



Ver	Indegree	Out.
1	1	2
2	2	1
3	0	1
4	2	1

Dom (R) = {1, 2, 3, 4}

Range (R) = {1, 2, 4}

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Path of length n

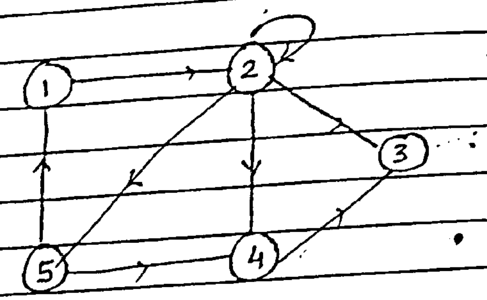
let R be a relation on set A
A path of length n in R from a to b is finite sequence

$\pi = a, x_1, x_2 \ldots x_{n-1}\, b$

where $aRx_1,\ x_1Rx_2 \ldots x_{n-1}Rb$

i.e. $(a, x_1) \in R$
$(x_1, x_2) \in R$ --
$(x_{n-1}, b) \in R$

e.g. For the following diographs determine the path of length 1&2, ~~3,4~~.



$R = \{(1,2)(2,2)(2,3)(2,4)(2,5)(4,3)$
$(5,1)(5,4)\}$

$\{\pi = 1,2,2 \longrightarrow 1\ R^2\ 2$
$\pi = 1,2\ 3 \longrightarrow 1\ R^2\ 3$
$\pi = 1\ 2\ 4 \longrightarrow 1\ R^2\ 4$
$\pi = 1\ 2\ 5 \longrightarrow 1\ R^2\ 5$

$\pi = 2\ 2\ 2$ $\quad 2\ R^2\ 2$
$\pi = 1\ 2\ 5$ $\quad 1\ R^2\ 2$
$\pi = 2\ 4\ 3$ $\quad 2\ R^2\ 3$
$\pi = 2\ 5\ 2$ $\quad 2\ R^2\ 4$
$\pi =$ $\quad 2\ R^2\ 5$

A path that begins and end with same vertex are called cycle.

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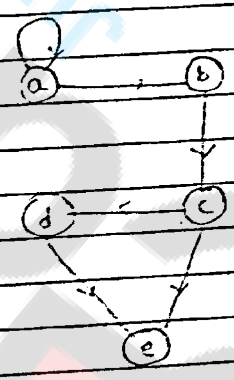


$A = \{a, b, c, d, e\}$

$R = \{(a,a)(a,b)(b,c)(c,d)(d,e)(c,e)$

Find (1) R^2
(2) Draw the diographs of R^2
(3) R^∞

To determine R^2. we need to find path of length 2 in diagraph for R.

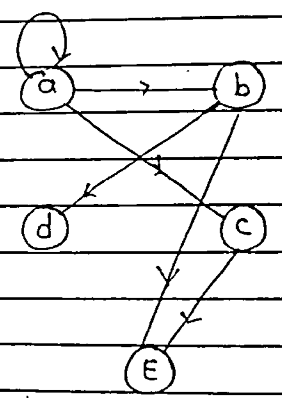
$a R^2 a$: a-a-a
$a R^2 b$: a-a-b
$a R^2 c$: a-b-c

$b R^2 d$: b-c-d
$b R^2 e$: b-c-e

$c R^2 e$: c-d-e

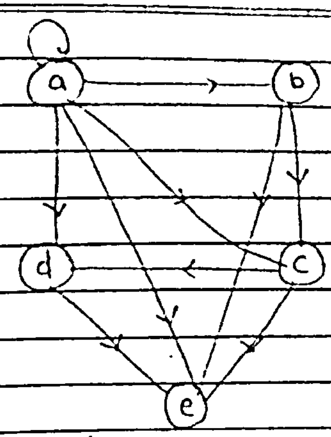
(1) $R^2 = \{(a,a),(a,b),(a,c),(b,d),(b,e),(c,e)\}$

(2)



diagraph of R^2.

(3) $R^\infty = \{(a,a)(a,b)(a,c)(a,d)(a,e),(b,c)(b,d)(b,e)(c,d)(c,e)(d,e)\}$



diagraph of R^∞

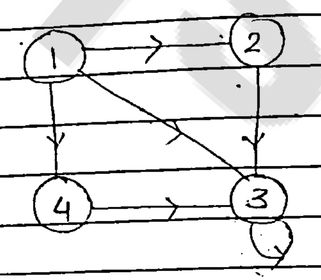
- If n is fix positive integer, if we define relation R^n on A.
- are follows $x R^n y$ means that there is a path of length n. from x to y in R
- we may also define the relation R^∞ on A in the following may $x R^\infty y$ means that there is a path from x to y.

- The length of such path is dependent of the value of x & y.

- R^∞ is also called as connectivity relation for R.

i.e. R^n specifies all the paths length n
R^∞ ~~tells~~ has pairs (x,y) where we x can reach from x to y.

We can find paths of length n by using the matrix.



$1 R^2 3$: 1-2-3

$2 R^2 3$: 2-3-3

$3 R^2 3$: 3-3-3

$4 R^2 3$: 4-3-3

$R = \{(1,2)(1,3)(1,4)(2,3)(3,3)(4,3)\}$

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 6 & 1 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

(0010) x (1000)

To find R^2.

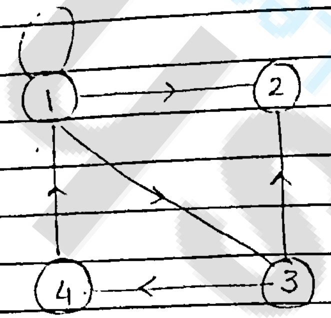
we first find : $M_{R^2} = (M_R)^2 = M_R \odot M_R$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M_{R^2} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

$R^2 = \{(1,3)(2,3)(3,3)(4,3)\}$

$(M_{R^2})^2 = M_{R^4}$



Find ① R^{∞}
② length 3 & 4.

$R^{\infty} = \{(1,1),(1,2),(1,3),(1,4),(3,1)(3,2),(3,4),(4,1),(4,2)(4,3)\}$

~~Properties of~~

Operations on Relation

① The compliementary relation on R is denoted by $\overline{R}$.
It is relation from $a \rightarrow b$ such that if $(a,b) \in R \Rightarrow (a,b) \notin \overline{R}$

② Intersection

$R \cap S$ means if $(a,b) \in R$ & $(a,b) \in S$ then $(a,b) \in R \cap S$

③ Union

$R \cup S$ means if $(a,b) \in R$ OR $(a,b) \in S$ then $(a,b) \in R \cup S$.

④ R^{-1}

The relation R^{-1} define in following way.
If $R : A \longrightarrow B$
$R^{-1} : B \longrightarrow A$
$(a,b) \in R \Rightarrow (b,a) \in R^{-1}$

Eg. Let $A = \{1,2,3\}$
let R and S be relations on A define by the matrices

$$M_R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_S = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Find - ① $M_{R \cup S}$
② $R \cap S$
③ R^{-1}
④ $\overline{R}$

① $M_{R \cup S} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 3 & 0 & 1 & 1 \end{array}$

$R \cup S = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$

② $M_{R \cap S} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & & 0 \end{bmatrix} \end{array}$

$R \cap S = \{(1,3)\ (2,2)\}$

③ $M_{R^{-1}} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 \end{array}$

$R^{-1} = \{(1,1)\ (2,2),(3,1)\ (3,2)\}$

$R = \{(1,1)(1,3)\ (2,2)\ (2,3)\}$

④ $M_{\bar{R}} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 \end{array}$

$\bar{R} = \{(1,2)(2,1)(3,1)(3,2)(3,3)\}$

Warshalls Algorithm

let R be set on Relation A

P Set $A = \{a_1, a_2, a_3 --- a_n\}$

If $x_1, x_2 --- x_n$ is a path in P then any vertices other than x_1 & x_m are called as interior vertices of the path.

The $1 \leq k < n$. we define a boolean matrix W_K as follows.

W_K has 1 in the position i.j if and only if there is a path from a_i to a_j whose interior vertices can come from the set $\{a, a_2, --- a_K\}$

Step 1 1) let R be relation on A.

Step 1 $W_0 = M_R$ the matrix for relation

Step 2 For $k = 1$ to n, where n is the number of element in A.

1st transfer to W_K ~~to~~ all the 1's in W_{K-1}.

Step 3 List the location $P_1, P_2, ---$ in the col. k of W_{K-1}. where ever the entry is one. and, location $q_1, q_2, \ldots$ in row k of W_{k-1} wherever the entry is 1.

Step 4 Put 1's in all the position p_i, q_j of W_k if they don't exit.

Eg. Find R^∞ by using Warshall's algorithm for the following relation whose diagram A

$A = \{1, 2, 3, 4\}$

$R = \{(1,2), (2,3), (3,4), (2,1)\}$

$$M_R = W_0 = \begin{array}{c} \\ \to 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$n = 4$

$1 \le k < 4$

$k = 1$

col $p \to 2$

row $q \to 2$

$M_R = (2,2)$

$$W_1 = \begin{array}{c} \\ 1 \\ \to 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$k = 2$

col $p \to 1, 2$

row $q \to 1, 2, 3$

$(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)$

$$W_2 = \begin{array}{c} \\ 1 \\ 2 \\ \to 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$k = 3$

col $p \to 1, 2$

row $q \to 4$

$(1,4)\ (2,4)$

$$W_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \to 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$k = 4$

col, $p \to 1, 2, 3$

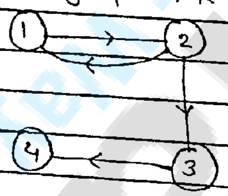
row, $q \to$ —

$W_4 = W_3$. [$\because$ q position are nu...]

$$W_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$R^\infty = \{((1,1), (1,2)(1,3)(1,4)$
$(2,1)\ (2,2)\ (2,3)(2,4)\ (3,4)\}$

Diagraph of R



$R^\infty = \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)$
$(2,3)(2,4)(3,4)\}$

(2) Find R^∞ for the following matrix given by,

$$M_R = W_0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$1 \le k \le 5$

$k = 1$

Col, $p \to 1, 2$

row, $q \to 1, 2$

$M_R = (1,1)\ (1,2)\ (2,1)(2,2)$

$$W_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$K = 2$

$p \to 1, 2$

$q \to 1, 2$

$M_R = (1,1)\ (1,2)\ (2,1)\ (2,2)$

	1	2	3	4	5
1	1	1	0	0	0
W_2 = 2	1	1	0	0	0
→ 3	0	0	1	1	0
→ 4	0	0	1	1	0
5	0	0	0	0	1

$K = 3$

$p \to 3, 4$

$q \to 3, 4.$

$M_R = (3,3)(3,4)(4,3),(4,4)$

$W_3 = W_2.$

$K = 4$

$p \to 3, 4.$

$q \to 3, 4.$

$M_R = (3,3)(3,4)(4,3)(4,4)$

$W_4 = W_3$

$K = 5$

$p \to 5$

$q \to 5.$

$M_R = (5,5).$

$W_5 = W_4$

Closure

$A = \{1, 2, 3, 4\}$

$R = \{(1,1)(1,2)(2,2)(2,3)(3,4)\}$

Test weather the following relation is ~~Reflexive~~, Reflective, Symmetic and Transitive and also find their respective closure.

i) Test for Reflective.

As $(3,3)$ and $(3,4) \notin R$.

The relation is not ~~refac~~ reflective.

The Reflective closure of R. $= \{(1,1)(1,2)(2,2)(2,3)(3,4)(3,3)(3,4)\}$

ii) Test for symmetric relation.

As $(1,2) \in R$, $(2,1) \notin R$.

$(2,3) \in R$, $(3,2) \notin R$.

$(3,4) \in R$, $(4,3) \notin R$.

The relation is not symmetric.

The ~~Reflective~~ Symmetric closure $= \{(1,1)(1,2)(2,2)(2,3)(3,4)(2,1)(3,2)(4,3)\}$

Test for transitive closure.

As $(2,3), (3,4) \in R$ but $(2,4) \notin R$

The relation is not transitive.

For transitive closure is we use warshall's algorithm.

$R = \{(1,1)(1,2)(2,2)(2,3)(3,4)\}$

$$W_0 = M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline \to 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$1 \leq k \leq 4$

$k = 1$

col $\to$ $p \to 1$

$q \to 1,2$

$M_R = (1,1)\ (1,2)$.

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ \to 2 & 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$k = 2$

col, $p \to 1,2$

row, $q \to 2,3$

$M_R = \{(1,2)(1,3)(2,2)(2,3)\}$.

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 \\ \to 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$k = 3$

col, $\to$ $p \to 1,2$

row, $q \to 4$.

$M_R = \{(1,4)\ (2,4)\}$

$$W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$k = 4$

col $\to$ $p \to 1,2,3$.

row $\to$ $q \to 0$.

$\therefore W_3 = W_4$

because q positions are null.

$aRb \equiv (a,b) \in R$.

A relation that is reflective, symmetric and transitive is called an equivalence relation.

Let $A = \mathbb{Z}$ set of instruction.
let R be relation on A defined by
aRb iff test weather $a \leq b$.

As given in the defination if $a \leq b$ we have $(a,b) \in R$.

① Test for reflectivity.

We know that the relation is reflective $(x,x) \in R$ if $x \in A$.

$\therefore$ x will be reflective if $x \leq x \ \forall x \in \mathbb{Z}$ which is true.

$\therefore$ R is reflective.

② Test for symmetric relation.

let $(x,y) \in R$
$x \leq y$
$\not\Rightarrow y \leq x$.
$\therefore (y,x) \notin R$

$\therefore$ R is not symmetric relation.

Greatest Common Divisor.

③ Test for transitive relation.

Let $(x,y) \in R$, & $(y,z) \in R$.

$\Rightarrow \{x \leq y$ & $y \leq z$.

$\Rightarrow x \leq z \quad \forall x,y,z \in \mathbb{Z}$

$\therefore$ R is transitive relation.

$\therefore$ R is not an equivalence relation.
bcoz R is not symmetric.

Eg. let A is set of all positive integer
$A = \mathbb{Z}^+ = \{0, 1, 2, 3, \ldots\}$

aRb iff $GCD(a,b) = 1$

Determine weather the Relation R is reflective, irreflective, symmectric, asymmectric, antisymmectric, transitive and give explanation

$\longrightarrow$

We know that $GCD(a,b) = 1$ when a & b are relatively prime.
i.e. the only common divisor betn then is 1.

$R = \{(1,1)(1,2)(1,3) \ldots$
$(2,1)(2,3), \ldots \}$

(i) Test for Reflective:

As $GCD(x,x) = x \neq 1$

$\therefore (x,x) \notin R$

$\therefore$ R is not ~~a~~ Reflective.

(ii) Test For IRReflective

As ~~(1,1)~~ $GCD(1,1) = 1$

$(1,1) \in R$

$\therefore$ R is not irreflective.

(iii) Symmetric

let $(x,y) \in R$

$\therefore GCD(x,y)$ ~~$\in R$~~ $= 1$

$\because GCD(y,x) = 1$

$(y,x) \in R$

$\therefore$ R is symmetric relation.

(iv) Asymmectic

let $(x,y) \in R$

$\therefore GCD(x,y) = 1$

$\therefore GCD(y,x) = 1$

$\therefore (y,x) \in R$.

$\therefore$ R is not asymmectric

(v) Antisymmectric

The relation is not antisymmetric bcoz $(2,3) \in R$ & $(3,2) \in R$.
but $2 \neq 3$ $(\because a = b)$

$\therefore$ R is not Antisymmetric

(vi) Transitive.

let $(x,y) \in R$ & $(y,z) \in R$

$GCD(x,y) = 1$, $GCD(y,z) = 1 \not\Rightarrow GCD(x,z) = 1$

Eg.

$GCD(2,3) = 1$ $2 R 3$ $(2,3) \in R$

$GCD(3,4) = 1$ $(3,4) \in R$

$GCD(2,4)$ ~~$\neq 1$~~ $= 2 \neq 1$ $(2,4) \notin R$.

$\therefore$ R is not Transitive.

Eg. A is set of integer.
aRb iff $|a-b| = 4$

$R = \{(0,4)(4,0)(1,5)(5,1),(2,6)$
$(6,2)(0,-4)(4,-0)(-1,-5)(-5,-1)\ldots$

① Reflective

As $|x-x| = 0$

$\therefore x \not R x \quad / \quad (x,x) \notin R$

$\forall x \in Z.$

$\therefore$ R is not Reflective.

② Irreflective.

And for the same reason R is irreflective.

③ Symmectric.

let $(x,y) \in R$

$|x-y| = 4$

$\Rightarrow |y-x| = 4$

$\Rightarrow (y,x) \in R.$

$\therefore$ R is symmectric.

④.

As it is asymmectric

It can not be asymmectric & antisymmetric

⑤ transitive

let $(x,y) \in R$ & $(y,z) \in R$

$\Rightarrow |x-y| = 4$, & $|y-z| = 4 \not\Rightarrow |x-z| = 4$

Eg. $(2,6) \in R$ $(6,2) \in R$ but $(2,2) \notin R$

Eg. Test weather

$A = \{1,2,3,4\}$

R is relation on A.

x divides y . $x|y$ $(x,y) \in R$ iff x divides y.

$R = \{(1,1)(1,2)(1,3)(1,4)$
$(2,2)(2,4)(3,3)(4,4)\}$

① Reflexive.

The relation is reflexive all the $(1,1)(2,2)(3,3)(4,4) \in R.$

② It is not irreflexive for same reason.

③ Symmetric

$(1,2) \in R$ but $(2,1) \notin R.$

$\therefore$ It is not symmetric.

$\therefore$ It is also not asymmetric

(4) It is an antisymmetric.

$(x,y) \in R$ & $(y,x) \in R$.

$x = y$.

(5) Transitive.

let $(x,y) \in R$ & $(y,z) \in R$.

x divides y $\quad$ y divides z

$\therefore 4$ divides 24

$24 = 4 \times \underset{k}{6}$.

$\therefore y = k_1 x$ $\quad$ $z = k_2 y$

$\because z = k_2 y$.

$= k_2 (k_1 x)$

$z = k_1 k_2 x$ $\quad \because (k_3 = k_1 k_2)$

$z = x k_3$

$\therefore x$ ~~dive~~divides z.

$\therefore$ R is transitive Relation.

Let R be relation on A $B \subseteq A$, then the Restriction of R to B is $R \cap (B \times B)$

(1) Find restriction of R to B (ii) $R(a)$ (iii) $R(b), R(c)$

let $A = \{a, b, c, d, e, f\}$

$R = \{(a,a) (a,c) (b,c) (a,e)$
$(b,e) (c,e)\}$

Find $R(T)$ where $T = \{a,b,c\}$

$\rightarrow B = \{a,b,c\}$

(1) Restriction of R to B

$R \cap (B \times B)$

$R \cap \{(a,a) (a,b) (a,c)$
$(b,a) (b,b) (b,c)$
$(c,a) (c,b) (c,c)\}$

Restriction of R to B.

$= \{(a,a), (a,c), (b,c)\}$

(2) Find $R(a) = \{a, c, e\}$

$R(b) = \{c, e\}$

$R(c) = \{e\}$

$R(d) = \phi$

$F(1) \cdot \{a, c, c'\}$

Equivalance

Let R be an equivalance relation on set A.

The set of all equivalance classes of a/c is called as the quotient sent and it is denoted by A/R

The equivalance classes are nothing but R relative A elements of s.

(1) $A = \{1, 2, 3, 4\}$

$R = \{(1,1)(1,2)(2,1)(2,2)(3,4)(4,3)(3,3)(4,4)\}$

Verify R is an equivalance relation. Determine its equivolance classes & also Find the quotiant set A/R.

i) Reflexive : $(x, x) \in R$ $\forall x \in A$

i.e. $(1,1)(2,2)(3,3)(4,4) \in R$.

ii) Symmetric

if (x,y) we have $(y,x) \in R$

iii) Transitive

$(1,1) \to (1,2), (1,1)$

$(2,1) \to (1,1), (1,2)$

$\therefore$ The relation is Equivalence relation

$R(1) = [1] = \{1, 2\}$

$R(2) = [2] = \{1, 2\}$

$R(3) = [3] = \{3, 4\}$

$R(4) = [4] = \{3, 4\}$

$[1] = [2]$

$[3] = [4]$

$A/R = \{\{1,2\}, \{3,4\}\} = \{[1], [3]\}$

② Test weather the following relations are Reflective, IRReflective, Symmetric, Asymmetric, Antisymmetric, transitive.

$A = Z$

aRb iff $a \leq b+1$

i) ~~RF~~ Reflective.

$(a,b) \in R$ iff. $a \leq b+1$.

$a \leq a+1$ $\quad\therefore a \leq a+1$

$a - a \leq 1$ $\quad$ or $(a,a) \in R$

$0 \leq 1$ true.

$\therefore$ It is Reflective.

ii) Let $(a,b) \in R$.

$a \leq b+1$

$a = 1$

$b = 3$.

$(a,b) \in R$.

$1 \leq 3+1$

but $(b,a) \notin R$

$\because 3 \nleq 1+1$

It is not symmetric.

iii) Asymmetric

Not a asymmetric.

Not an antisymmetric.

iv) Transitive

$R(8,9)$ iff $8 \leq 9+1$

$R(9,10)$ iff $9 \leq 10+1$

$R(8,10)$ iff $8 \leq 10+1$

~~At~~

$R(10,9)$ $\quad 10 \leq 9+1$ ✓

$R(9,8)$ $\quad 9 \leq 8+1$ ✓

$R(10,8)$ $\quad 10 \leq 8+1$ ✗

Not an transitive relation.

② aRb iff $a^2 - b^2 = 4$

Composition of a Relation:

A, B, C are sets.

$R: A \longrightarrow B$

$S: B \rightarrow C$

We can define a new relation, a composition of R & S.

Given by $SOR : A \longrightarrow C$

$S(R(a))$

The relation is defined in a such a way that, $a \in A, b \in B, c \in C$ then $(a,c) \in SOR$ if their exit some $b \in B$ such that $(a,b) \in R$, $(b;c) \in S$

e.g. (1) Let R be the relation

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Determine: (i) SoR (ii) RoS

$S: A \rightarrow A$, $S: B \rightarrow C$

$R: A \rightarrow A$

* let us find SoR, for this we find $M_{SOR} = M_R \odot M_S$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{SOR} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$M_{SOR} = \{(1,1)(1,3)(2,1)(2,2)(2,3)(3,2)(3,3)\}$

$M_{ROS} = M_S \odot M_R$.

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 \end{array}$$

$ROS = \{(1,1)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

(2) $A = \{1, 2, 3, 4, 5\}$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

1. R^{-1}
2. $\overline{R}$
3. $R \cup S$
4. $R \cap S$
5. $R \circ S$
6. $S \circ R$
7. R^{∞}
8. R^T
9. Diagraph R
10. Indegree outdegree of all ever
11. Restriction of R to subset of A
12. Relative sets of elements.
13. —"— subsets.
14. Reflexive clousure.
15. Symmetric
16. Transitive.

Rules of Inferena for propositional calculus.

1) Addition $\quad \dfrac{P}{\therefore P \vee Q}$

2) Conjunction $\quad Q$, $\therefore P \wedge Q$

3) Simplification $\quad \dfrac{P \wedge Q}{P}$

4) Modus Pones $\quad \dfrac{P \quad P \to Q}{\therefore Q}$

5) Modus Tollens $\quad \dfrac{\sim Q \quad P \to Q}{\sim P}$

6) Disjunctive Syllogism : $\quad \dfrac{\sim P \quad P \vee Q}{\therefore Q}$

7) Hypothetical Syllogism : $\quad \dfrac{P \to Q \quad Q \to R}{\therefore P \to R.}$

8) Constructive Dilemma : $\quad (P \to Q) \wedge (R \to S)$

$$\dfrac{P \vee R}{Q \vee S}$$

9) Destructive Dilemma

$(P \to Q) \wedge (R \to S)$
$\sim Q \vee \sim S$
$\sim P \vee \sim R$

Que: Can we ~~concluld~~ colude S, From the following premises.

i) $P \to Q$
ii) $P \to R$
iii) $\sim(Q \wedge R)$
iv) $S \vee P$

$\Longrightarrow$

i)	$P \to Q$	Premise 1.
ii)	$P \to R$	Premise 2
iii)	$(P \to Q) \wedge (P \to R)$	Conjunction
iv)	$\sim(Q \wedge R)$	Premise 3
v)	$\sim Q \vee \sim R$	Demorgan's law.
vi)	$\sim P \vee P$	
vii)	$\sim P$	
viii)	$S \vee P$	
ix)	S.	

Let S be the set of integers.

$A = S \times S$

Define the relation R on A as

$\therefore aRb$ iff $a \equiv b \bmod 5$ [a is conguent to ~~m~~ b mod 5]

Show that R is an Equivalance relation & determin A/R.

Test for Equivalance.

An equivalance ~~relation~~ refation is reflective, Symmectric & transitive.

① Test for reflexive

* Given $(a, b) \in R$ iff $a \equiv b \bmod 5$.

Test $(a, a) \in R$.

$a \equiv a \bmod 5$ which is true.

R is Reflexive.

② Test for symmetric.

* Let $(a, b) \in R$.

$\Rightarrow a \equiv b \bmod 5$

a & b give same ~~then~~ rem. when divide 5.

b & a give same ~~then~~ rem. when divide 5

$b \equiv a \bmod 5$.

$(b, a) \in R$.

③ Test for transitive :

$(a,b) \in R$ & $(b,c) \in R$

$a \equiv b \mod 5$ & $b \equiv c \mod 5$

a & b give same rem. when divedes / b & c give same rem. when divid 5.

→ a, b, c have the same rem. when divide 5.

$a \equiv c \mod 5$. ∴ Transitive

It is equivalance relation.

Que. Find ~~question~~ quotiant set A/R

For this we fine equivalance classes.

$[0] = \{0, 5, 10, 15, ---- , -5, -10, -15, ---\}$

$[1] = \{1, 6, 11, --- - , -1, -6, -11, ---\}$

$[2] = \{2, 7, 12, --- , -2, -7, ---\}$

$[3] = \{3, 8, 13, --- , -3, -8, -13, --\}$

$[4] = \{4, 9, 14, -- , -4, -9, -14- --\}$

∴ The quotient set is

$A/R = \{[0], [1], [2], [3], [4]\}$

① Test weather R is an Equivalance relation and find equivalance classes. and also find A/R

$A = \{1, 2, 3, 4\}$

$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,3)\ (3,4)\ (4,3)\ (4,4)\}$

→ ① Test weather relation is ~~Symmetric~~ Reflexive.

As $(x,x) \in R$ for $\forall x \in A$

∴ R is reflexive.

② Symmetric.

As if $(x,y) \in R$ we have $(y,x) \in R$ for $\forall$ all the $(x,y) \in R$.

~~th.~~

∴ The R is symmetric.

③ Transitivety.

if 1^{st} & last matrix are same then relation is transitive (we can also find by warshall's)

$(1,1) \in R$ < $(1,2)$, $(1,1)$

$(a,b) \in R$ & $(b,c) \in R$

we have $(a,c) \in R$

∴ The relation is transitive.

∴ The relation is equivalance.

$[1] = \{1,2\}$

$[2] = \{1,2\}$

$[3] = \{3,4\}$

$[4] = \{3,4\}$

$A/R = \{[1], [3]\}$

OR

$A/R = \{\{1,2\}, \{3,4\}\}$

Note :- we can check the relation is transitive by warshall's algo. also.

∴ If W_0 & $M_R = W_n$ (the last warshall's matrix) then relation is transitive.

Test weather the following relation on A is an equivalance relation and if it is, find A/R.

where $A = S \times S$ S: set of integers.

28/01/2019

Partial order relation.

A relation R on set A is called as partial order relation ~~is~~ if R is as ~~partial or~~ reflective, Antisymmetric, transitive and (A,R) is called POSET.

Q.1 If $A = \{1,2,3,4,12\}$ & Relation R defined as aRb iff a divide b.

Prove that R is partial order or (A,R) is poset.

→ Reflective :-

To prove that R is Reflective.

from R we can see that $(x,x) \in R$ ∀ $x \in A$.

because x/x always we must have xRx or $(x,x) \in R$.

So R is Reflective.

Antisymmetric

To prove R is antisymmetric.
let $(x,y) \in R$ & $(y,x) \in R$.

$\Rightarrow$ x divides y & y divides x.

$\Rightarrow x = y$.

$\therefore$ The Relation R is antisymmetric.

Transitive

To prove that R is transitive.
let $(x,y) \in R$, & $(y,z) \in R$.

x divides y & y divides z.

$\Rightarrow y = x k_1$ $\quad z = y k_2$. for some integer k_1 & k_2.

$\Rightarrow z = x \cdot k_1 k_2$.
$z = x \cdot k_3$

$\Rightarrow$ x divides z.
$(x,z) \in R$.

Hasse diagram

We can draw the Hasse diagram for every POSET.

Step 1 :- Draw the diagraph of R.

Step 2 :- As the Relation is always reflective (P.O) so drop the self loops in the diagram.

Step 3 :- Since the relation is always transitive delete the ~~eges of~~ edges of type of (x,z) whenever we have the edge (x,y) & (y,z)

Step 4 Do not show the arrows along the edges of the diagraph & use only dot for vertex. do not draw circle.

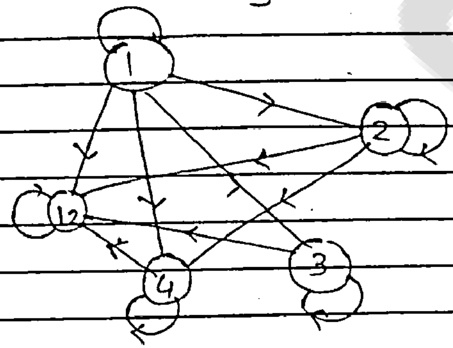
Step 5 Use some order. write smaller element on lower level.

Note :-
If $a, b \in R$ then we normally used $a \leq b$.
Thus. If I have $(10,5) \in R$.

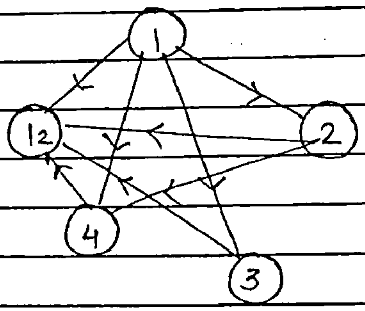
$A = \{1, 2, 3, 4, 12\}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,12)$
$(2,2)\ (2,4)\ (2,12)$
$(3,3)\ (3,12)$
$(4,4)\ (4,12)$
$(12,12)\}$

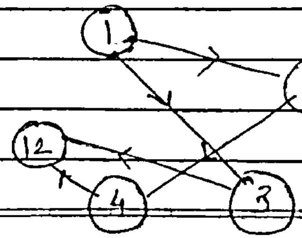
Step 1



Step 2 Remove loops.

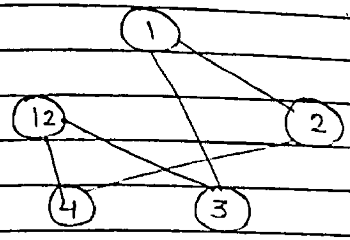


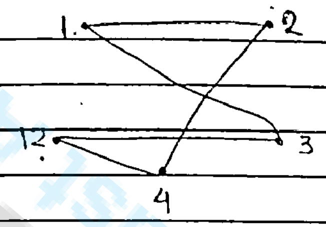
Step 3



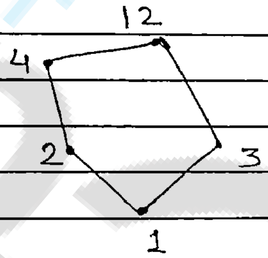
If I have
$(a,b)\ (b,c)$
then don't draw.
(a,c).

Step 4 Remove the arrows.





Step 5



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① If A is the power set of S. define the relation subset relation on A. Prove that it is partial order. Also draw the hasse diagram for same.

$\Rightarrow$

$A = P(S)$

$P(S) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$(x,y) \in R$ iff $x \leq y$.

① Reflective.

$x \leq x$ is true

$(x,x) \in R$

R is reflective

(ii) Antisymmetric

Let $(x,y) \in R$ & $(y,x) \in R$

$\Rightarrow x \leq y$ & $y \leq x$

$\Rightarrow x = y$

$\therefore$ R is Antisymmetric.

(iii) Transitive

let $(x,y) \in R$ & $(y,z) \in R$.

$x \leq y$ & $y \leq z$.

$x \leq z$ $\therefore$ Transitive

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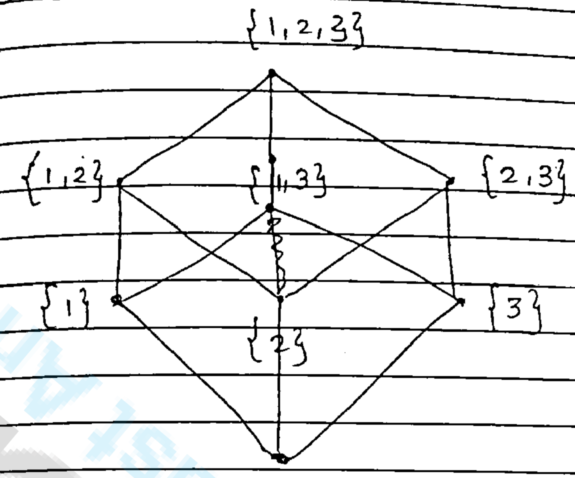
Hence R is partial order on power set of A.



② Prove that the following relation is partial order on set A.

$A = \{1, 2, 3, 4, 8, 12\}$

Given by aRb iff a divides b & draw hasse diagram.

Also find minimal & maximal elements least & greatest element if they exit.

$\Rightarrow$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,8)\ (1,12)$
$(2,2)\ (2,4)\ (2,8)\ (2,12)$
$(3,3)\ (3,12)\ (4,4)\ (4,18)\ (4,12)$
$(8,8)\ (12,12)\}$

(i) Reflective

x divides x $\forall x \in A$

$\Rightarrow (x,x) \in R$ $\forall x \in A$

$\therefore R$ is reflexive

(ii) Antisymmetric

let $(x,y) \in R$ & $(y,x) \in R$

x divides y & y divides x

$\Rightarrow x = y$

$\therefore R$ is antisymmetric

(iii) Transitive

let $(x,y) \in R$ & $(y,z) \in R$

x divides y y divides z

$y = x \cdot k_1$ $z = y \cdot k_2$ (where k_1 & k_2 int)

$z = x \cdot k_1 \times k_2$

$z = x \cdot k_3$

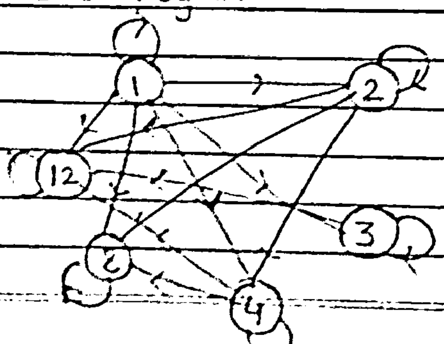
$\therefore x$ divides z.

$\therefore R$ is transitive

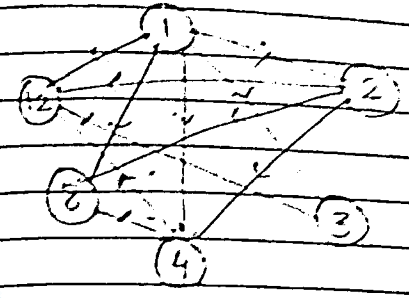
$\therefore R$ is Partial order.

Hasse diagram.

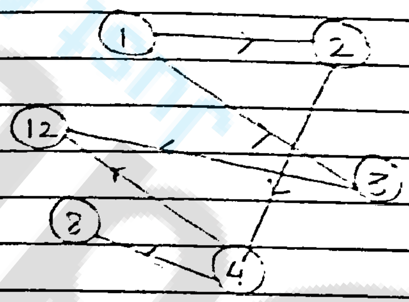
Step (i)



Step 2 : Remove loops.



Step 3 Remove transitive



$1 < 2 < 4 < 12$

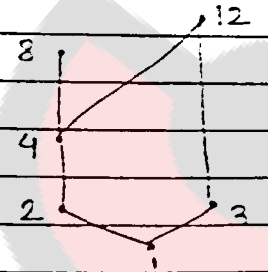
$1 < 4 < 8$

$3 < 12$

$4 < 12$

$4 < 8$

Step 4 :- Remove circle & arrow



Note : We can find relation for Hasse diagram

(i) The minimal element is 1.

(ii) There are 2 maximal element 8 & 12.

(iii) 1 is the least element & their is no greatest element.

(1) A is the set of Integer,
R is a relation on A defined by
aRb iff $a \equiv b \mod 7$

Test equivalence & find A/R.

→

Equivalence : Reflexive, Symmetric, transitive.

→

(1) Reflexive
Given $(a,b) \in R$ iff $a \equiv b \mod 7$.

Test $(a,a) \in R$

② $A = \{1, 2, 3, 4, 5\}$

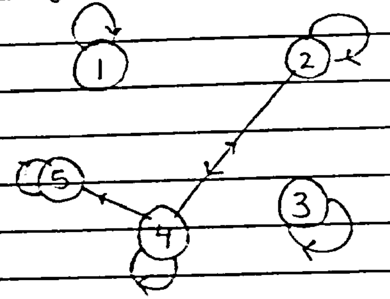
$$M_R = \begin{array}{c|ccccc|c} & 1 & 2 & 3 & 4 & 5 & \text{Outdeg} \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 1 & 0 & 2 \\ 3 & 0 & 0 & 1 & 0 & 0 & 1 \\ 4 & 0 & 1 & 0 & 1 & 1 & 3 \\ 5 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline \text{Indegree} & 1 & 2 & 1 & 2 & 2 & \end{array}$$

$\Rightarrow$

① write R

$R = \{(1,1), (2,2), (2,4), (3,3), (4,2), (4,4), (4,5), (5,5)\}$

② Draw diograph.



③ Find indegree & outdegree of each vertex.

Vertex	Indegree.	Outdegree
1	1	1
2	2	2
3	1	1
4	2	3
5	2	1

④ Dom(R), Range(R)

$Dom(R) = \{1, 2, 3, 4, 5\}$

$Range(R) = \{1, 2, 3, 4, 5\}$

⑤ $R(2)$, $R(4)$, $R(S)$ when $S = \{3, 5\}$

$\rightarrow$

$R(2) = \{$

6) Test if the relation is,

① Reflective

let $(a,a) \in R$. ~~for~~ $\forall a \in A$.

i.e. $(1,1)\ (2,2)\ (3,3)\ (4,4) \in R$.

$\therefore$ The relation is reflexive.

② Symmetric

if $(x,y) \in R$ then we have $(y,x) \in R$.

In this R, $(4,5) \in R$ & $(5,4) \notin R$

$\therefore$ R is not Symmetric.

(3) Transitive

if $(a,b) \in R$ & $(b,c) \in R$.
then $(a,c) \in R$

i)	$(1,1) \in R$		
ii)	$(2,2) \in R$,	$(2,4) \in R$	$(2,4) \in R$
iii)	$(2,4) \in R$,	$(4,2) \in R$	$(2,2) \in R$
		$(4,4) \in R$	$(2,4) \in R$
iv)	$(3,3) \in R$		
v)	$(4,2) \in R$	$(2,2) \in R$	$(4,2) \in R$
		$(2,4) \in R$	$(4,4) \in R$.
vi)	$(4,4) \in R$	$(4,2) \in R$	$(4,2) \in R$.
vii)	$(4,5) \in R$	$(5,5) \in R$	$(4,5) \in R$.
viii)	$(5,5) \in R$	$(5,5) \in R$	$(5,5) \in R$.

$\therefore$ R is transitive.

(4) Antisymmetric.

$(a,b) \in R$ & $(b,a) \in R$.
we have $a=b$.
i.e. $(1,1) \in R$ & $(1,1) \in R$.
& $1=1$.

$\therefore$ Relation is not Antisymmetric.

bcoz $(2,4) \in R$ & $(4,2) \in R$
are present.

(5) Asymmetric

$(a,b) \in R$ & $(b,a) \notin R$.

$\therefore$ The relation is not Asymmetric.

(6) Irreflexive.
$(a,a) \notin R$.
$\therefore$ The relation is not irreflexive.

(7) Equality.

$(2,4) \in R$
$\therefore$ The R is not Equality.

(8) Inequality
$(1,1), (2,2) \in R$.
$\therefore$ The relation is not inequality relation

(9) Partial order.
(Reflective, Symmetic & Antisymmetric)
The relation is not Antisymmetric
& symmetric.
$\therefore$ The R is not partial oreder
on set A.

If only one element is found then it is called least or greatest/minimal & maximal element

Q.1 $A = \{1, 2, 3, 4, 6, 8, 12\}$

$\{A, \text{divides } b\}$

$x R y$ iff x divides y.

$R: \{(1,1)(1,2)(1,3)(1,4)(1,6)(1,8)(1,12)$
$(2,2)(2,4)(2,6)(2,8)(2,12)$
$(3,3)(3,6)(3,12)$
$(4,4)(4,8)(4,12)(6,6)(6,12)$
$(8,8)(12,12)\}$

i) Find it is POSET & draw hasse diagram
ii) Find minimal & maximal elements.
iii) Find least element, greatest if it exist
~~It is POSET & Find~~
iv) If $b_1 = \{2, 3, 6\}$

$b_2 = \{2, 4\}$

Find the upper bound & lower bound G.L.B., L.U.B of b_1 & b_2.

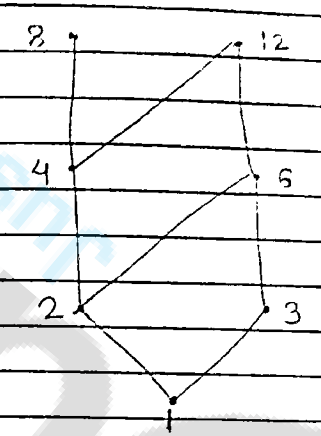
Minimal : No element is less than that element

Maximal element : No element greater than that element

It is POSET

Because it is Reflexive, Antisymmetric and transitive

Hasse diagram:-



(i) Minimal element is 1

(ii) There are two maximal elements 8 & 12.
There is no greatest element.

(iii) least element is 1.

(iv) $UB(2) = \{2, 6, 4, 8, 12\}$

$UB(3) = \{3, 6, 12\}$

$UB(6) = \{6, 12\}$

Upper bound of B1 are $\{6, 12\}$
~~lower bound of~~ B1 = 6.
least Upper bound

$B1 = \{2, 3, 6\}$

lower bound of $(2) = \{2, 1\}$

$(3) = \{1, 3\}$

$(6) = \{6, 2, 3, 1\}$

L.B. B1 = 1.

Greatest lower bound of b1 = 1.

$B2 = \{2, 4\}$

→ $UB(2) = \{2, 4, 6, 8, 12\}$

$UB(4) = \{4, 8, 12\}$

$UB(B2) = \{4, 8, 12\}$

Least Upper bound = 4.

→ $LB(2) = \{2, 1\}$

$LB(4) = \{4, 2, 1\}$

$LB(B2) = \{2, 1\}$

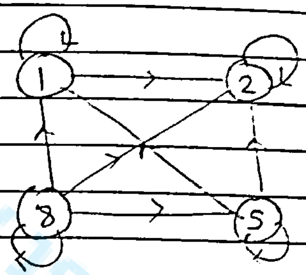
Greatest lower bound = 2.

(2) let $A = \{1, 2, 5, 8\}$

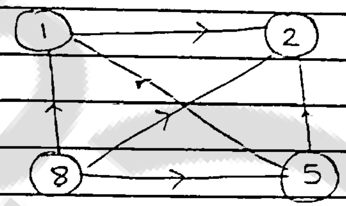
aRb iff $a \geq b$.

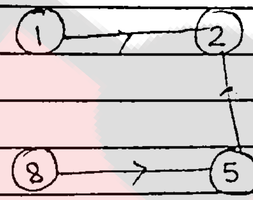
→ $R = \{(1,1)\ (2,1)\ (2,2)\ (5,1)\ (5,2)\ (5,5)\ (8,1)\ (8,2)\ (8,5)\ (8,8)\}$

Digraph

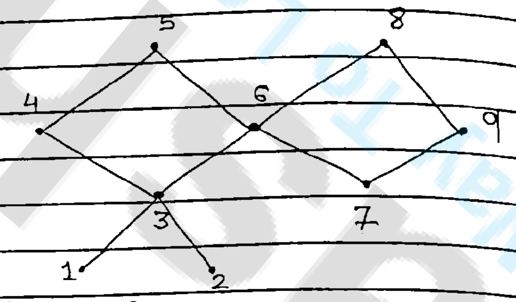


Hasse diagram:





This type of hasse diagram is called chain & the partial order (PO) is called as linear order.



$B = \{3, 4, 5\}$

Find the all Upper & lower bounds of B.

GLB & LUB of B if they exist.

→ $B = \{3, 4, 5\}$

$UB(3) = \{3, 4, 6, 8, 5\}$

$UB(4) = \{4, 5\}$

$UB(5) = \{5\}$

UB of B = 5.

Least Upper Bound of B = 5.

→ $B = \{3, 4, 5\}$

$LB(3) = \{1, 2, 3\}$

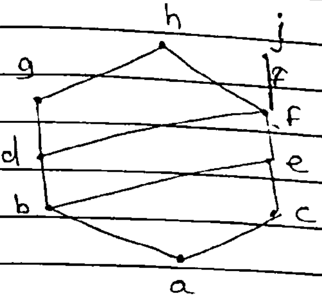
$LB(4) = \{4, 3, 2, 1\}$

$LB(5) = \{5, 4, 6, 7, 1, 2, 3\}$

LB of $B = \{1, 2, 3\}$

Greatest lower bound = 3.

(a)



~~$\{a, c, d, f\}$~~ $B_1 = \{a, b, c\}$

$B_2 = \{j, h\}$

⟹ $UB(a) = \{a, b, e, d, f, g, h, j\}$

$UB(B) = \{b, e, d, f, g, h, j\}$

$UB(c) = \{c, e, f, j, h\}$

$UB(B_1) = \{e, f, j\}$

Least Upper bound = e.

→ $LB(a) = \{a\}$

$LB(b) = \{a, b\}$

$LB(c) = \{a, c\}$

G.L.B = a

⟹ $B_2 = \{j, h\}$

$UB(j) = \{j\}$

$UB(h) = \{h\}$

$L.U.B = \phi$

$LB(i) = \{f, e, c, d, b, a\}$

$LB(h) = \{g, d, b, f, e, c, a\}$

$LB(B_2) = \{a, b, c, d, e, f\}$

Greatest Lower bound = f.

Q.1 Which element of the poset $(\{2,4,5,10,12,20,25\}, |)$ ① are maximal and which are minimal? → a divides b

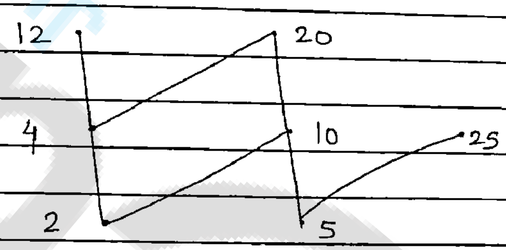
② Find least & Greatest element if they exit.

③ Find Upper bound, lower bound, glb, lub if they exist of B_1 & B_2.

$B_1 = \{4, 10\}$ $B_2 = \{5, 10, 25\}$

→ $\{(2,2)\ (2,4)\ (2,10)\ (2,12)\ (2,20)\ (4,12)\ (4,20)\ (5,10)\ (5,20)\ (5,25)\ (10,20)\}$

Hasse diagram :-



① Maximal elements are 12, 20 & 25.

② Minimal elements are 2 & 5.

③ Least & Greatest element are does not exist.

To Find Upper Bound of B_1. We Find UB 4 & 10.

③ ① $B_1 = \{4, 10\}$

→ $UB(4) = \{4, 12, 20\}$

$UB(10) = \{10, 20\}$

$UB(B_1) = \{20\}$

~~Greatest~~ least Upper bound = 20 //

→ $LB(4) = \{1, 2\}$

$LB(10) = \{1, 2, 5\}$

$LB(B_1) = \{2\}$

~~least~~ Gretest Lower bound = $\{2\}$ //

(ii) $B_2 = \{5, 10, 25\}$

→ $UB(5) = \{5, 10, 20, 25\}$

$UB(10) = \{10, 20\}$

$UB(25) = \{25\}$

There is no Upper bound of B_2.

~~UB(B1) =~~

~~LUB =~~

→ $LB(5) = \{5\}$

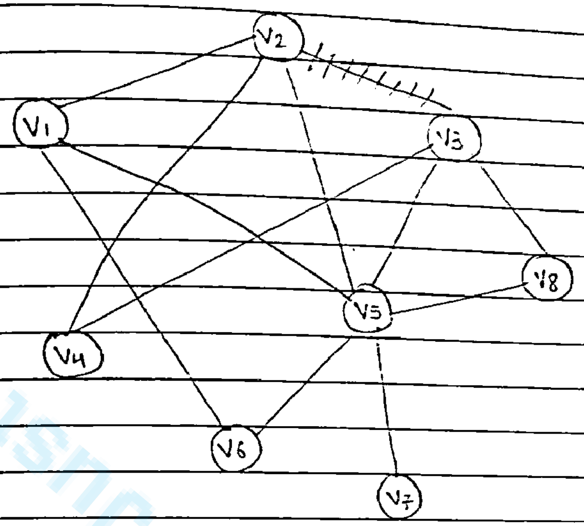
$LB(10) = \{2, 5, 10\}$

$LB(25) = \{5, 25\}$

$LB(B_2) = \{5\}$

$GLB = 5$ //

Find adjoncency matrix and Adjoncency List for the following graph.



Adjoncency matrix.

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
V_1	0	1	0	0	1	1	0	0
V_2	1	0	0	1	1	0	0	0
V_3	0	0	0	$\cancel{0}1$	$\cancel{0}1$	0	0	1
V_4	0	$\cancel{0}1$	$\cancel{0}1$	0	0	0	0	0
V_5	1	1	1	0	0	1	1	1
V_6	1	0	0	0	1	0	0	0
V_7	0	0	0	0	1	0	0	0
V_8	0	0	1	0	1	0	0	0

Adjoncency list.

$V_1 \rightarrow V_2 \rightarrow V_5 \rightarrow V_6$

$V_2 \rightarrow V_1 \rightarrow V_4 \rightarrow V_5$

$V_3 \rightarrow V_4 \rightarrow V_5 \rightarrow V_8$.

$V_4 \rightarrow V_2 \rightarrow V_3$

$V_5 \rightarrow V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_6 \rightarrow V_7 \rightarrow V_8$

$V_6 \rightarrow V_1 \rightarrow V_5$

$V_7 \rightarrow V_5$

$V_8 \rightarrow V_3 \rightarrow V_5$

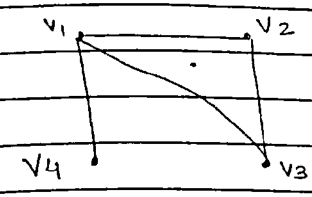
Euler Path and Circuit.

A path in graph G is called as Euler path if it includes every edge of the graph exactly ones.

Euler's circuit

Euler path which is circuit that is which start & end at same vertex and at every edge its travel exactly ones.

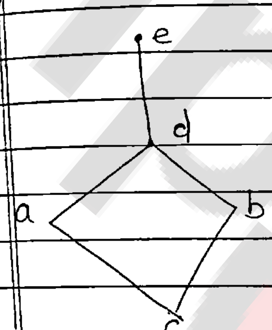
Find Euler's circuit & Euler's path if it's exist.

① 

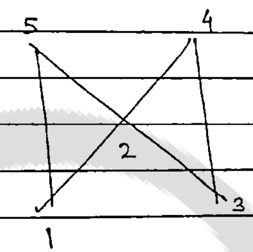
$V_1\ V_2\ V_3\ V_1\ V_4$.

$V_1\ V_2\ V_3\ V_1\ V_4$

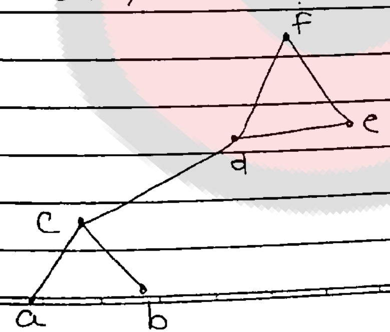
There is not ~~Elu~~ Euler's path.

② G1 

Euler's path circuit ≠.

G2 

Euler's path & circuit.



No euler's path.

Fleury's Algorithm

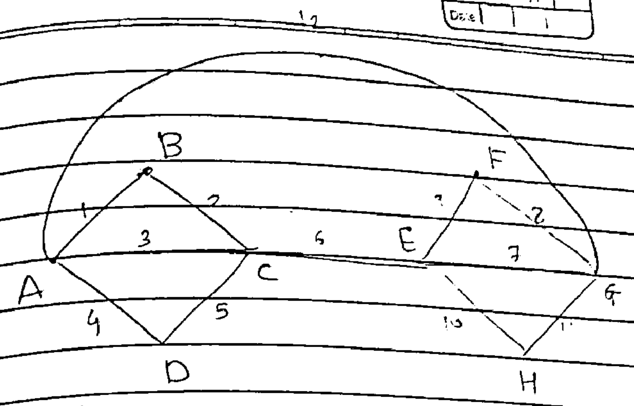
Let $G = (V, E, \gamma)$ be a connected graph with each vertex of even degree.

Step 1. Select a member v of V as the beginning vertex for the circuit. Let $\pi : v$ designated the beginning of the path to be constructed.

Step 2 Suppose that $\pi, v, u, \ldots, w$ has been constructed thus far. If at w there is only one edge $[w, z]$, extend π to $\pi, v, u, \ldots, w, z$. Delete $[w, z]$ from E and w from V. If at w there are several edges, choose one that is not a bridge to the remaining graph, say $[w, z]$. Extend π to $\pi, v, u, \ldots, w, z$ and delete $[w, z]$ from E.

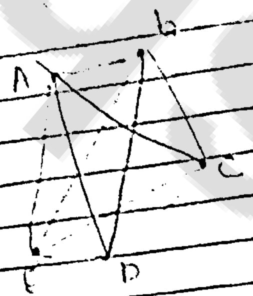
Step 3: Repeat step 2 until no edges remain in E.

End of Algorithm



Path	Delete
π : A	{AB}
π : AB	{BC}
π = ABC	{CA}
π : ABCA	{AD}
π : ABCAD	{DC}
π : ABCADC	{CE}
π : ABCADCE	{EG}
π : ABCADCEG	{GH}
π : ABCADCEGH	{HE}
π : ABCADCEGHE	{EF}
π : ABCADCEGHEF	{FG}
π : ABCADCEGHEFG	{GA}

π : ABCADCEGHEFGA.



Path	Delete
π : A	{AB}
π : AB	{BE}
ABE	{EA}
ABEA	{AC}
ABEAC	{CB}
ABEACB	{BD}
ABEACBD	{DC}
ABEACBDC	{DE}
CE	

Euler circuit

Theorem 1.

a) If a graph G has a vertex of odd degree, there can be no Euler circuit in G

b) If G is a connected graph & every vertex has even degree, then there is an Euler circuit in G

	Yes	No
Euler's circuit	If every vertex is even degree	At least one vertex is of odd degree
Euler's path	If there are 2 vertices of odd degree it starts at 1st odd degree & end at other	More then 2 vertices of odd degree.

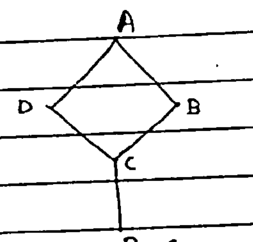
Theorem 2:-

a) If a graph G has more than two vertices of odd degree, then there can be no Euler's path in G

b) If G is connected and has exactly two vertices of odd degree, there is an Euler's path in G. Any Euler path in G must begin at one vertex of odd degree & end at the other.

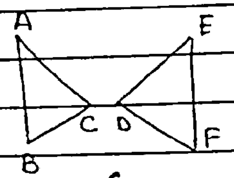
Hamiltonion path
is a path that contains each vertex exactly once.

→ Hamiltonion circuit:
Each vertex exactly once & path starts and ends at the same vertex. except 1^{st} & last.

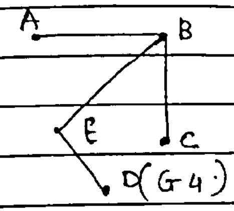


G1

G2



G3



(G4)

G1 and G2 have hamiltonion path.

G_1 = ~~A~~ ECDAB
G_2 = ABCD → path
ABCDA → circuit.

A compe~~l~~ete graph kn on n vertices has a hamiltonions circuit. Further you can visit other verticies ~~&~~ sequentially in desire order.

Note: complete graph is a graph in which each vertices in joined to every other remaining vertices.

For kn

No. of Vertex.	No. of edges.
1	0
2	1
3	3
4	6
5	10

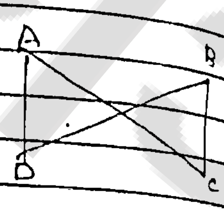
let G be the connected graph of n vertices, $n \geq 2$ & no loops and multiple edges then G has hamiltonian circuit if for any two vertices U & V of G that are not adjacent, the degree of (U) + deg. (V) $\geq n$

Here $n = 4$

A & B not adjacent

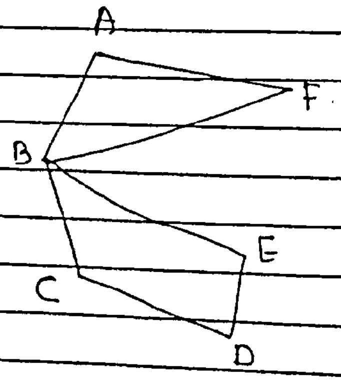
deg. (A) = 2

deg. (B) = 2.



deg (A) + deg. (B) = $2+2 \geq 4$.

Test weather the following graph are Eurel's path, circuit & hamiltoin path, circuit.



If A has a hamiltonian circuit with each vertex has degree $\geq n/2$.

- If the no. of edges in G is M & no. of vertices n
- A has a hamiltonian circuit then

$$m \geq \frac{1}{2}(n^2 - 3n + 6)$$

The converse of the above theorem need not be true.