2nd UT

Page No.
Date 05/03/2019.

Recurence Relations.

let $\{a_0, a_1, --- a_n\}$

$a_n = F\{a_{n-1}, a_{n-2}, --- a_4, a_n\}$ for any n.

for eg:

The fibbonaci series

1, 1, 2, 3, 5, ---

$a_n = a_{n-1} + a_{n-2}$.

$a_0 = 1$, $a_1 = 1$

$a_2 = a_1 + a_0$.

$a_3 = a_{3-1} + a_{3-2}$

$a_3 = a_2 + a_1$

$a_3 = 2 + 1 = 3$

$a_2 = a_1 + a_0$

$= 1 + 1$

$= 2$.

Q.1 Determine weather a sequence a_n is the solution of a recourrence relation.

$a_n = 2a_{n-1} - a_{n-2}$ for $n = 2, 3, ---$

$a_n = 5$ for every non negative integer n.

→

$a_n = 5$

$a_{n-1} = 5$

$a_{n-2} = 5$.

$5 = 2 \times 5 - 5$. $\therefore 5 = 5$.

L.H.S = R.H.S

∴ Given relation is recourrence relation.

Q.2 Test weather a_n is a sol^n of the recurrence relation,

$a_n = 2a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, ---$

$a_n = 2n$ for every non -ve integer.

→

$a_n = 2a_{n-1} - a_{n-2}$ —(1)

Given , $a_n = 2n$

$a_{n-1} = 2(n-1)$

$a_{n-2} = 2(n-2)$

~~$a_n = 2n$~~.

$C_1 \cdot 3^{n-1} = C_1 \cdot 3^n \cdot 3^{-1} = \frac{C_1 \cdot 3^n}{3}$

L.H.S : $a_n = 2n$

R.H.S : $2a_{n-1} - a_{n-2}$

$= \cancel{2 \times (} 2 \times 2(n-1) - 2(n-2)$

$= 4n - 4 - 2n + 4$

$= 2n \qquad = L.H.S.$

$\therefore a_n = 2n$ is a solⁿ of given difference eqⁿ

Q.1 # Solution of Recurrence relation.

$$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} = f(n)$$

is $3^n + 4^n + 2$ $\quad f(n) = 6$ for all n.

determine C_0, C_1 & C_2.

→

Given,

The solution is

$$a_n = 3^n + 4^n + 2$$

$$\therefore a_{n-1} = 3^{n-1} + 4^{n-1} + 2$$

$$a_{n-2} = 3^{n-2} + 4^{n-2} + 2$$

$$f(n) = 6$$

$$C_0[3^n + 4^n + 2] + C_1[3^{n-1} + 4^{n-1} + 2] + C_2[3^{n-2} + 4^{n-2} + 2] = 6$$

Collecting the coefficient of 3^n & 4^n and constant at both side we get,

$$C_0 3^n + C_0 \cdot 4^n + 2C_0 + C_1 3^{n-1} + C_1 \cdot 4^{n-1} + 2 \cdot C_1 + C_2 \cdot 3^{n-2} + C_2 \cdot 4^{n-2} + 2 \cdot C_2 = 6$$

$$3^n\left(C_0 + \frac{C_1}{3} + \frac{C_2}{3^2}\right) + 4^n\left(C_0 + \frac{C_1}{4} + \frac{C_2}{4^2}\right) + 2(C_0 + C_1 + C_2) = 0 \times 3^n + 0 \times 4^n + 6$$

Comparing coeffient of L.H.S & R.H.S. for 3^n, 4^n & constant term. we get 3 eqⁿ.

$$C_0 + \frac{C_1}{3} + \frac{C_2}{3^2} = 0 \quad — (2)$$

$$C_0 + \frac{C_1}{4} + \frac{C_2}{4^2} = 0 \quad — (3)$$

$$2(C_0 + C_1 + C_2) = 6 \quad — (4)$$

$$C_0 + C_1 + C_2 = 3.$$

$$C_0 = 3 - C_1 - C_2$$

From ②,

$$(3-c_1-c_2)+\frac{c_1}{3}+\frac{c_2}{9}=0 \quad —⑤$$

From ③,

$$(5-c_1-c_2)+\frac{c_1}{4}+\frac{c_2}{16}=0 \quad —⑥$$

From ⑤, (L.C.M)

$$\frac{9(3-c_1-c_2)+3c_1+c_2}{9}=0.$$

$$27-9c_1-9c_2+3c_1+c_2=0$$

$$-6c_1-8c_2+27=0 \quad —⑦$$

From ⑥,

$$\frac{16(3-c_1-c_2)+4c_1+c_2=0}{16}$$

$$48-16c_1-16c_2+4c_1+c_2=0.$$

$$-12c_1-15c_2+48=0. \quad —⑧$$

3 by 3 determinent.

$$-48 \quad 27 \quad 21$$

$$-12c_1-15c_2+48=0$$

$$-6c_1-8c_2+27=0. \times 2.$$

$$-12c_1-15c_2+48=0.$$

$$-\;\;-12c_1-16c_2+54=0$$

$$+ \quad + \quad -$$

$$c_2-6=0$$

$$\boxed{c_2=6}$$

$$-6\times c_1-8\times 6+27=0.$$

$$-6c_1-48+27=0.$$

$$-6c_1-21=0.$$

$$-6c_1=21.$$

$$c_1=\frac{21}{6}=\frac{-7}{2}//$$

$$\boxed{c_1=-7/2}$$

From ④,

$$c_0+c_1+c_2=3.$$

$$c_0=3-c_1-c_2$$

$$=3+\frac{7}{2}-6$$

$$=\frac{6+7-12}{2}$$

$$\boxed{c_0=\frac{1}{2}}$$

$$\Delta c_1 = \begin{vmatrix} 0 & 1/2 & 1/4 \\ 0 & 1/3 & 1/9 \\ 8 & 1 & 1 \end{vmatrix}$$

2/9

$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} = f(n)$$

Sol^n is : $a_n = 2^n + 3^n + 5$

$f(n) = 40 \quad \forall n$

Find c_1, c_2, c_3.

$$c_0(2^n + 2^n + 5) + c_1(2^{n-1} + 3^{n-1} + 5) + c_2(2^{n-2} + 3^{n-2} + 5) = 40.$$

$$2^n\left(c_0 + \frac{c_1}{2} + \frac{c_2}{2^2}\right) + 3^n\left(c_0 + \frac{c_1}{3} + \frac{c_2}{3^2}\right) + 5(c_0 + c_1 + c_2) = 40.$$

$$c_0 + \frac{c_1}{2} + \frac{c_2}{4} = 0 \quad -(1)$$

$$c_0 + \frac{c_1}{3} + \frac{c_2}{3^2} = 0 \quad -(2)$$

$$c_0 + c_1 + c_2 = 8 \quad -(3)$$

$4 - 20 + 24$

$4 + 4$

$= 8$

$$\Delta = \begin{vmatrix} 1 & 1/2 & 1/4 \\ 1 & 1/3 & 1/9 \\ 1 & 1 & 1 \end{vmatrix} \qquad -1/18$$

$$\Delta = 1\left(\frac{1}{3} - \frac{1}{9}\right) - \frac{1}{2}\left(1 - \frac{1}{9}\right) + \frac{1}{4}\left(1 - \frac{1}{3}\right)$$

$$= 1\left(\frac{3-1}{9}\right) - \frac{1}{2}\left(\frac{9-1}{9}\right) + \frac{1}{4}\left(\frac{3-1}{3}\right)$$

$$= 1\left(\frac{2}{9}\right) - \frac{1}{2}\left(\frac{8}{9}\right) + \frac{1}{4}\left(\frac{2}{3}\right)$$

$$= \frac{2}{9} - \frac{8}{18} + \frac{2}{12}$$

$$= \frac{8 - 16 + 6}{36} = \frac{-8+6}{36} = \frac{-2}{36} = \frac{-1}{18}$$

$$\Delta c_1 = \begin{vmatrix} 0 & 1/2 & 1/4 \\ 0 & 1/3 & 1/9 \\ 8 & 1 & 1 \end{vmatrix}$$

$$= 0 - \frac{1}{2}\left(0 - \frac{8}{9}\right) + \frac{1}{4}\left(\frac{-8}{3}\right)$$

$$= \frac{4}{9} - \frac{2}{3}$$

$$= \frac{4-6}{9}$$

$$= \frac{-2}{9}$$

$$\Delta C_2 = \begin{vmatrix} 1 & 0 & 1/4 \\ 1 & 0 & 1/9 \\ 1 & 8 & 1 \end{vmatrix}$$

$$= 1\left(-\frac{2}{9}\right) + \frac{1}{4}\left(8\right)$$

$$= \frac{-8}{9} + 2$$

$$= \frac{-8+18}{9}$$

$$= \frac{10}{9}$$

$$\Delta C_3 = \begin{vmatrix} 1 & 1/2 & 0 \\ 1 & 1/3 & 0 \\ 1 & 1 & 8 \end{vmatrix}$$

$$= 1\left(\frac{8}{3} - 0\right) - \frac{1}{2}\left(8 - 0\right) + 0$$

$$= 1\left(\frac{8}{3}\right) - \frac{1}{2}\left(8\right)$$

$$= \frac{8}{3} - 4$$

$$= \frac{8-12}{3} = \frac{-4}{3}$$

$$\frac{\Delta C_1}{\Delta} = \frac{-2}{9} \times -18 = 4$$

$$\frac{\Delta C_2}{\Delta} = \frac{10}{9} \times -18 = -20$$

$$\frac{\Delta C_3}{\Delta} = \frac{-4}{3} \times -18 = 24$$

Date: 06/03/2019

Recurrence Relation

Use back tracking method to find the sequence $\{a_n\}$ as a solution of the recurrence relation.

$$a_n = a_{n-1} + 3, \quad a_1 = 2.$$

$$a_n = a_{n-1} + 3 \quad \text{--- (1)}$$

Put $n = n-1$ in eq^n (1)

$$a_{n-1} = a_{n-2} + 3 \quad \text{--- (2)}$$

eq^n (1) becomes,

$$a_n = [a_{n-2} + 3] + 3.$$

$$a_n = a_{n-2} + 3 + 3 \quad \text{--- (3)}$$

If we put $n = n-1$ in eq^n (2), we get.

$$a_{n-2} = a_{n-3} + 3.$$

We use it in eq^n (3),

$$a_n = [a_{n-3} + 3] + 3 + 3.$$

$$a_n = a_{n-3} + 3 + 3 + 3.$$

$$a_n = a_{n-3} + 3 \times (3)$$

$a_n = a_{n-k} + k \cdot 3$

Substitute $k = n-1$ in the above eqn we get.

$a_n = a_{n-(n-1)} + (n-1) \times 3$

$a_n = a_1 + 3(n-1)$

$a_n = 3(n-1) + 2$

These is the solution for given Ag recurrence solution.

Q.2 Find the soln of recurrence relation $b_n = 2b_{n-1} + 1$, $b_1 = 7$

$b_n = 2b_{n-1} + 1$ — (1)

$b_{n-1} = 2b_{n-2} + 1$ — (2)

$b_n = 2[2b_{n-2} + 1] + 1$

$b_n = 2^2 b_{n-2} + 2 + 1$ — (3)

$b_n = 2^2[2b_{n-3} + 1] 2 + 1$

$b_n = 2^3 \cdot b_{n-3} + 2^2 + 2 + 1$

$b_n = 2^k b_{n-k} + 2^{k-1} + 2^{k-2} \cdots + 2 + 1$

Put $k = n-1$

$b_n = 2^{n-1} b_{n-(n-1)} + 2^{n-2} + 2^{n-3} + 2 + 1$

$b_n = 2^{n-1} b_1 + \boxed{2^{n-2} + 2^{n-3} + \ldots + 2 + 1}$

$a + ar + ar^2 + \ldots + ar^{n-1}$

$a[1 + r + r^2 + r^3 + \ldots + r^{n-1}] = \frac{a[r^n - 1]}{r-1}$

$1 + r + r^2 \ldots + r^{n-1} = \frac{r^n - 1}{r-1}$

$1 + 2 + 2^2 + \ldots + 2^{n-2} = \frac{2^{n-1} - 1}{2-1}$

$b_n = 2^{n-1} \cdot b_1 + \frac{2^{n-1} - 1}{1}$

$b_n = 2^{n-1} \times 7 + 2^{n-1} - 1$

$= 8 \cdot 2^{n-1} - 1$

$b_n = 2^3 \cdot 2^{n-1} - 1$

$= 2^{n-1+3} - 1$

$b_n = 2^{n+2} - 1$

State the problems of towers of Hanoi. Find the recurrence relation and solve it.

let n be the no. of disk

~~#~~ let a_n is the no. of moves required to move no. of disk.

For $n = 1$ $a_n = 1$ ~~move~~ | move

~~from~~	~~to~~	from	to
n	a_n		
$n = 2$	∴ 3	A	C
		B	B
		C	B

~~$t = 1$~~

n	a_n	
1	1	a_1
2	3	$a_2 = 1 + 1 + 1 = 2a_1 + 1$
3	7	$a_3 = 3 + 3 + 1 =$ $a_2 + a_2 + 1 = 2a_2 + 1$
4	15	$a_4 = 7 + 7 + 1 = a_3 + a_3 + 1$ $= 2a_3 + 1$

For n disk we get,

$n = 2a_{n-1}+1$

this the recurrence relⁿ for the tower of hanoi problem.

We can solve this by back tracking method.

$a_n = 2a_{n-1} + 1 \quad —(1) \quad a_1 = 1$

$a_{n-1} = 2a_{n-2} + 1 \quad —(2)$

eqⁿ (I) become,

$a_n = [2a_{n-2}+1]+1$

$a_n = 2^2 \{a_{n-2} + 2 + 1$

$a_n = 2^2[2a_{n-3}+1]+2+1$

$a_n = 2^3 a_{n-3} + 2^2 + 2 + 1$

$a_n = 2^k a_{n-k} + 2^{k-1} + 2^{k-2} \ldots + 1.$

Put, $k = n-1$

$a_n = 2^{n-1} \cdot a_1 + 2^{n-1-1} + 2^{n-1-2} \ldots + 2 + 1$

$a_n = 2^{n-1} \times 1 + 2^{n-2} + 2^{n-3} \ldots + 2 + 1$

$a_n = 2^{n-1} + 2^{n-2} + \ldots + 2 + 1.$

$a_n = \frac{2^n - 1}{2-1}$

$\boxed{a_n = 2^n - 1}$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Date: 9/03/2019

Sr.	Nature of Charactristic Roots.	Homogeneous sol^n.
1	Real & distinct m_1, m_2	$a_n = A(m_1)^n + B(m_2)^n$
2.	Real & Repeated $m_1 = m_2 = m_3 = m$	$a_n = (An^2+Bn+c)(m)^n$
3.	Real, some are repeated & some are distinct $m_1 = m_2 = m, m_3$	$a_n = (An+B)(m)^n + C(m_3)^n$

Q ① Solve the Recurrence relation, $a_n = a_{n-1} + 2a_{n-2}$ with initial condition $a_0 = 2, a_1 = 7$.

→

as RHS of eq^n equal to zero we only have homogenuous eq^n

$$a_n - a_{n-1} - 2a_{n-2} = 0 \quad \text{—} ①$$

$$\begin{bmatrix} 1 & -1 & -2 & = 0 \end{bmatrix} \quad \text{write coef}^n.$$

$$m^2 - m - 2 = 0.$$

$$m_1 = 2 \quad OR \quad m = -1$$

Real & distinct,

$$a_n = A(2)^n + B(-1)^n \quad \text{—} ②.$$

Given initial conditions,

$a_0 = 2$

we put $n=0$ in eq^n ②.

$$a_0 = A(2)^0 + B(-1)^0$$

$$2 = A + B \quad \text{—} ③.$$

$a_1 = 7$.

we put $n=1$ in eq^n ②.

$$a_1 = A(2)^1 + B(-1)^1$$

$$7 = 2A - B \quad \text{—} ④$$

Solve eq^n ③ & ④,

$$A + \cancel{B} = 2$$

$$+ \; 2A - \cancel{B} = 7$$

$$3A = 9$$

$$\boxed{A = 3}$$

Put $A=3$ in eq^n ③,

$$A + B = 2$$

$$3 + B = 2$$

$$\boxed{B = -1}$$

Put the value in eq^n ②.

$$a_n = 3(2)^n - (-1)^n$$

2) Find the solⁿ of Recurrence relⁿ $a_n = 3a_{n-1} - 2a_{n-2}$ with initial condⁿ $a_1 = 5$, $a_2 = 3$.

$\rightarrow$

$$a_n - 3a_{n-1} - 2a_{n-2} = 0 \quad \text{—} (1)$$

$$m^2 - 3m - 2 = 0$$

$$m^2 - 2m - 1m - 2 = 0$$

$$m(m-2) - 1(m-2) = 0$$

$$(m-1)(m-2) = 0$$

$$\underline{m_1 = 1} \quad \text{OR} \quad \underline{m_2 = 2}$$

Real & distinct,

$$a_n = A(m_1)^n + B(m_2)^n$$

$$= A(1)^n + B(2)^n \quad \text{—} (2)$$

Given Initial condⁿ,

$a_1 = 5$.

$$a_1 = A(1)^1 + B(2)^1$$

$$a_1 = A + 2B$$

$$A + 2B = 5 \quad \text{—} (3)$$

$a_2 = 3$.

$$a_2 = A(1)^2 + B(2)^2$$

$$3 = A + 4B$$

$$A + 4B = 3 \quad \text{—} (4)$$

Solve eqⁿ (3) & (4),

$$\begin{array}{r} A + 4B = 3 \\ -\ A + 2B = 5 \\ \hline 2B = -2 \end{array}$$

$$\boxed{B = -1}$$

Put $B = -1$ in eqⁿ (3),

$$A + 2(-1) = 5$$

$$A - 2 = 5$$

$$A = 5 + 2$$

$$\boxed{A = 7}$$

Put in eqⁿ (2),

$$a_n = 7(1)^n + -1(2)^n$$

3) Find the recurrence relation.

$a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1$, $a_1 = 2$

$\longrightarrow$

$a_n - 6a_{n-1} + 9a_{n-2} = 0$ —(1)

$m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$\underline{m = 3}$ OR $\underline{m = 3}$.

As roots are repeated,

$a_n = (An+B)(m)^n$

$a_n = (An+B)(3)^n$ —(2)

Given initial condn,

$a_0 = 1$

$a_0 = (A0+B)(3)^0$

$1 = B$. —(3)

Put $a_1 = 2$.

$a_1 = (A1+B)(3)^1$

$2 = (A+B)(3)$

$A+B = 6$.

$\boxed{A+B=6}$ —(4)

(4) Solve the recurrence relalation,

$a_n + 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 0$.

$a_0 = 1,\ a_1 = -2,\ a_2 = 8$

$\longrightarrow$

$a_n + 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 0$.

$m^3 + 6m^2 + 12m + 8 = 0$. — (1)

$(m+2)(m^2+4m+4) = 0$.

$(m+2)(m^2+2m+2m+4) = 0$.

$(m+2)(m+2)(m+2) = 0$

$m_1 = -2,\ m_2 = -2,\ m_3 = -2$.

Real & Reapeated,

$a_n = (An^2+Bn+c)(m^n)$

$a_n = (An^2+Bn+c)(-2)^n$. — (2)

$a_0 = 1$ — $1 = C(-2)^0$. — (3)

$a_1 = -2$ — $-2 = (A+B+c)(-2)^1$

$+1 = A+B+C$. — (4)

$a_2 = 8$ — $\cancel{-1 = A+B+C}$ — (4).

$8 = (4A+2B+C)(-2)^2$

$4A+2B+C = 8/4$

$4A+2B+C = 2$ — (5).

$C = 1$ — (3)

$A+B+C = 1$ — (4) $\times 2$

$4A+2B+C = 2$ — (5)

$\cancel{2A+2B+2C = 1}$.

$\cancel{4A+2B+C = 2}$

$\cancel{-2A = -1}$

$\cancel{A = \frac{-1}{-2}}$

Put $C = 1$.

$A+B = 0$ — (6)

$4A+2B = 1$ — (7).

$2A+2B = 0$.

$4A+2B = 1$

$-2A = -1$

$\boxed{A = 1/2}$

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$$\boxed{B = -1/2}$$

$$a_n = \left(\frac{1}{2}n^2 - \frac{1}{2}n + 1\right)(-2)^n.$$

To Find the particulars solution ($a_n^{(P)}$)

R.H.S of eq^n not equal to zero, we have to see the type of R.H.S and then write the particular solution. The rules are given table in below,

Form of R.H.S	form of P.S.
① Constant	$a_n = A.$
② linear expression eg. $2n+5$	$a_n = An + B$
③ Quadratic expression eg. $3n^2 + 2n + 5$.	$a_n = An^2 + Bn + C$
④ constant × expo. 4^n, 42×4^n	$a_n = A \cdot 4^n$. Note: base of the expression is not characteristic root.
⑤ linear expression eg. $(2n+5)(4^n)$	$a_n = (An+B)4^n$.
⑥ Quad. expo. $(3n^2+2n+5) \times 4^n$.	$a_n = (An^2+Bn+C)4^n$

Total solution :-
Homogenous + Particular

① Find the particular sol^n of the recurrence relation.

$$a_n + 4a_{n-1} + 4a_{n-2} = n^2 - 3n + 5. \quad \text{—①}$$

→ To find the particular sol^n, consider the R.H.S.

$n^2 - 3n + 5$. it is quadratic eq^n.

∴ Particular sol^n is given by.

$$a_n^{(P)} = An^2 + Bn + C.$$

$$a_{n-1}^{(P)} = A(n-1)^2 + B(n-1) + C$$

$$a_{n-2}^{(P)} = A(n-2)^2 + B(n-2)^2 + C.$$

Use them in eq^n ①.

Page No.
Date

$\rightarrow (An^2+Bn+C)+4(A(n-1)^2+B(n-1)+C)$

$+4(A(n-2)^2+B(n-2)+C) = n^2-3n+5$

$\rightarrow 4[A(n^2-2n+1)+B(n-1)+C]$

$An^2-2An+A+Bn-B+C$

$4An^2+n4(-2A+B)+4(A-B+4)$

$\rightarrow 4(A(n^2-4n+4)+Bn-2B+C]$

$4An^2-16n+16+Bn-2B+C$

$4An^2+n(-16+B)+(16-2B+C)$

$n^2[A+4A+4A]+n[B+4(-2A+B+(-16+B)]$

$+C\{+4(A-B+C)+(16-2B+C)]$

$= n^2-3n+5.$

$A = 1/9$

$B = -13/27.$

$C = 7/9.$

Page No.
Date

② Find the particular solⁿ of the eqⁿ.

$\rightarrow a_n - 5a_{n-1} + 6a_{n-2} = 1. \quad -①$

As R.H.S is a constant, the particular solⁿ will be.

$a_n^{(P)} = A. \quad -②$

$a_{n-1}^{(P)} = A.$

$a_{n-2}^{(P)} = A.$

Substituting in eqⁿ ①,

$A - 5A + 6A = 1.$

$+2A = 1$

$A = +1/2$

Substituting in eqⁿ ②,

$a_n^{(P)} = 1/2.$

③ Find the particular sol^n for recurrence relation for,

$$a_n + 5a_{n-1} + 6a_{n-2} = 42 \times 4^n \quad \text{—}①.$$

$\longrightarrow$

$m^2 + 5m + 6 = 0$

~~$m^2 + 6m - 1m + 6 = 0$~~

~~$m(m+6) - m(m+6) = 0$~~

6 → ~~6 1~~, 2 3.

$(m+2)(m+3) = 0.$

$m_1 = -2, \quad m_2 = -3.$

Root $\neq 4$.

P.S is

$a_n^{(P)} = A \cdot 4^n.$

$a_{n-1}^{(P)} = A \cdot 4^{n-1}$

$a_{n-2}^{(P)} = A \cdot 4^{n-2}.$

Put the values in eq^n ①,

$A \cdot 4^n + 5A\,4^{n-1} + 6 \cdot A \cdot 4^{n-2} = 42 \times 4^n.$

for common term 4^n — $4^n \left[A + \frac{5A}{4} + \frac{6A}{4^2}\right] = 42 \times 4^n.$

$\therefore \; A + \frac{5A}{4} + \frac{6A}{4^2} = 42.$

S/w — System
— Appl^n (they are return to solve some specific task)
Eg: Medical store

9594980272.
7045564124

$\frac{16A + 20A + 6A}{16} = 42.$

$42A = 42 \times 16.$

$\boxed{A = 16.}$

$a_n^{(P)} = A \times 4^n$

$a_n^{(P)} = 16 \times 4^n.$

(I) Introduction to System s/w & OS.

Centralised OS.

Program execution Flow.