Computer Graphics

Q. I] $S_x = 1/2$ and $S_y = 1/3$

$A(4,1)$; $B(5,2)$; $C(4,3)$.

∴ We know that,

$$P' = P \cdot S$$

where $P = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix}$, $S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$ → scaling matrix

$$\therefore P' = \begin{bmatrix} 4 & 1 \\ 5 & 2 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1/3 \\ 5/2 & 2/3 \\ 2 & 1 \end{bmatrix}$$

Q. II] Apply the scaling transformation on triangle $A(10,10)$, $B(17,8)$ and $C(13,15)$ by keeping C fixed.

Solⁿ:- The transformation is performed in the foll steps:-

1) Translate the triangle so that it's centre coincides with the origin
2) Scale the triangle with respect to the origin.
3) Translate the triangle back to the original position with the centre still remaining at its same position.

1) $\therefore T_1 = \begin{bmatrix} \cancel{S_x}\,1 & 0 & 0 \\ 0 & \cancel{S_y}\,1 & 0 \\ -13 & -15 & 1 \end{bmatrix}$ Translating the triangle to ~~its~~ the origin

2) $S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Scaling the triangle about origin

3) $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 13 & 15 & 1 \end{bmatrix}$ Translating the triangle back to its original position.

$\therefore T = T_1 \cdot S \cdot T_2$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -13 & -15 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 13 & 15 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -13S_x & -15S_y & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 13 & 15 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -13S_x + 13 & -15S_y + 15 & 1 \end{bmatrix}$$

$\therefore$ we know that, $P' = P \cdot S$

$$\therefore \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 10 & 10 & 1 \\ 17 & 8 & 1 \\ 13 & 15 & 1 \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -13S_x + 13 & -15S_y + 15 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10S_x - 13S_x + 13 & 10S_y - 15S_y + 15 & 1 \\ 17S_x - 13S_x + 13 & 8S_y - 15S_y + 15 & 1 \\ \cancel{13S_x} - \cancel{13S_x} + 13 & \cancel{15S_y} - \cancel{15S_y} + 15 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} -3S_x + 13 & -5S_y + 15 & 1 \\ 4S_x + 13 & -7S_y + 15 & 1 \\ 13 & 15 & 1 \end{bmatrix}$$

Q.III] Using Bresenham's line Drawing algorithm, Rasterize the line between the endpoints (4,7) and (9,11)

Soln:- $(x_1, y_1) = (4,7)$; $(x_2, y_2) = (9,11)$

$\therefore \Delta x = x_2 - x_1 = 9 - 4 = \underline{5}$

$\Delta y = y_2 - y_1 = 11 - 7 = \underline{4}$

Initial descision parameter,

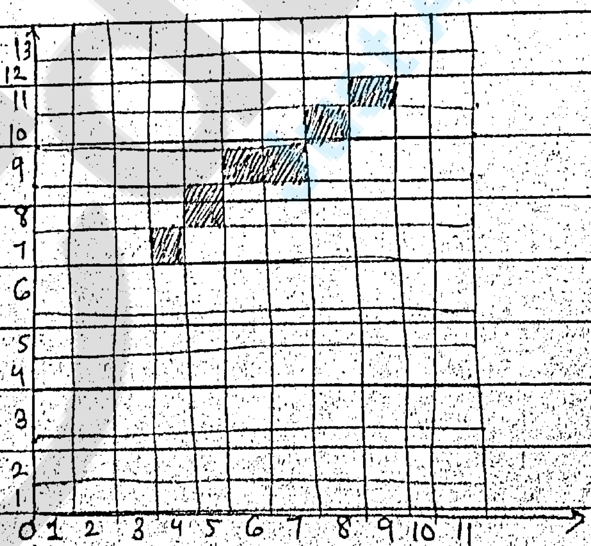
$\therefore e = 2\Delta y - \Delta x$

$= 2(4) - 5$

$= 8 - 5 = \underline{\underline{3}}$

Let the starting pixel point $(x,y) = (4,7)$

i	Plot	x	y	e
0	(4,7)	4	7	3
1	(5,8)	5	8	1
2	(6,9)	6	9	-1
3	(7,9)	7	9	~~21~~ 7
4	(8,10)	8	10	+5
5	(9,11)	9	~~9~~ 11	3



$\therefore e \geq 0$ ie. 3

1) $e = e + 2\Delta y - 2\Delta x$

$= 3 + 2(4) - 2(5)$

$= 3 + 8 - 10$

$= 11 - 10 = 1$

2) $e = e + 2\Delta y - 2\Delta x$

$= 1 + 2(4) - 2(5)$

$= 1 + 8 - 10$

$= 9 - 10 = -1$

3) $e = e + 2\Delta y - 2\Delta x$

$= -1 + 2(4) - 2(5)$

$= -1 + 8 - 10$

$= 7 - 10$

$= -3$

4) $e = e + 2\Delta y - 2\Delta x$

$= -3 + 2(4) - 2(5)$

$= -3 + 8 - 10$

$= 5 - 10 = -5$

3) $e = e + 2\Delta y$

$= -1 + 2(4) = -1 + 8$

$= 7$

4) $e = e + 2\Delta y - 2\Delta x$

$= 7 + 2(4) - 2(5)$

$= 7 + 8 - 10$

$= 15 - 10 = 5$

5) $e = e + 2\Delta y - 2\Delta x$

$= 5 + 2(4) - 2(5)$

$= 5 + 8 - 10$

$= 13 - 10 = 3$

Q-III) Plot a circle centered at (10,5) having a radius of 15 units

Soln:- Given:-

$r = 15$ units

$M = 1 - r$

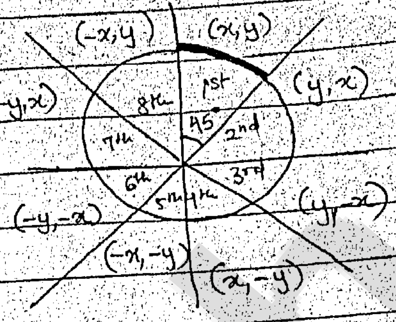
$= 1 - 15 = -14$

$(x_c, y_c) = (10, 5)$

$\therefore$ let $(x, y) = (0, 15)$

While plotting plot,

$(x + x_c, y + y_c)$



For the 1st octant, (x,y)

Plot	x	y	M
(10,20)	0	15	-14
(11,20)	1	15	-11
(12,20)	2	15	-6
(13,20)	3	15	1
(14,19)	4	14	-18
(15,19)	5	14	-7
(16,19)	6	14	6
(17,18)	7	13	-5
(18,18)	8	13	12
(19,17)	9	12	7
(20,16)	10	11	6

Similarly,

for plotting the symmetric points

i.e. octant 2,

plot $(x_c + y, y_c + x)$

Octant 3 : plot $(x_c + y, y_c - x)$

Octant 4 : plot $(x_c + x, y_c - y)$

Octant 5 : plot $(x_c - x, y_c - y)$

Octant 6 : plot $(x_c - y, y_c - x)$

Octant 7 : plot $(x_c - y, y_c + x)$

Octant 8 : plot $(x_c - x, y_c + y)$

FOR EDUCATIONAL USE

Scanned by CamScanner

Contrast Stretching function :-

$S = l * r \;;\; 0 \le r < r_1$

$= m(r-r_1)+v \;;\; r_1 \le r < r_2$

$= n(r-r_2)+w \;;\; r_2 \le r \le L-1$

Q.V] Apply the following transformations on the following 3 BPP image :-

a) Image negative.

b) Gray-level slicing with background when $r1 = 3$ & $r2 = 6$.

c) Thresholding with Threshold value $= 4$.

2	1	0	7	5
4	2	3	1	2
7	6	2	1	6
2	4	5	6	7
2	3	4	5	1

Soln a) No. of BPP = 3

No. of Gray levels = $2^3 = 8$

min level = 0 ; max level = 7

$\therefore$ Image Negative Transformation function,

$S = (L-1) - r$

$= (8-1) - r$

$= 7 - r$

$\therefore$ Resultant Image :

5	6	7	0	2
3	5	4	6	5
0	1	5	6	1
5	3	2	1	0
5	4	3	2	6

$r_1 = 3 ; r_2 = 6$

b) Gray-level slicing Transformation function with background :-

$$S = \begin{cases} L-1 & \text{if } a \le r \le b \\ = r & \text{o.w.} \end{cases}$$

$$\therefore S = \begin{cases} L-1 & \text{if } 3 \le r \le 6 \\ = r & \text{o.w.} \end{cases}$$

I/P gray level (r)	O/P gray level (s)
0	$S = 0$
1	$S = 1$
2	$S = 2$
3	$S = L-1 = 7$
4	$S = L-1 = 7$
5	$S = L-1 = 7$
6	$S = L-1 = 7$
7	$S = 7$

$\therefore$ Resultant Image :-

2	1	0	7	7
7	2	7	1	2
7	7	2	1	7
2	7	7	7	7
2	7	7	7	1

c] Threshhold value $(a) = 4$

$\therefore$ Thresholding function is given as,

$$S = 0 \quad ; \text{if } r \leq a$$
$$S = L-1 \quad ; \text{if } r > a$$

i.e. $S = 0 \quad ; \text{if } r \leq 4$

$S = 8-1 \quad ; \text{if } r > 4$

$= 7$

$\therefore$ Resultant Image :-

0	0	0	7	7
0	0	0	0	0
7	7	0	0	7
0	0	7	7	7
0	0	0	7	0

Q. VI] For the following eight bit image perform:-

a) Threshold, $T = 150$

b) Image Negative.

120	135	215	220	125
135	20	187	50	80
250	115	55	120	45
30	180	200	46	20
60	119	120	255	125

Soln:

No. of BPP = 8

No. of gray levels $= 2^8 = 256 = L$

min level = 0 ; max level = 255

a) Threshold value (a) = 150

$\therefore$ Thresholding function is as,

$$\left.\begin{array}{ll} S = 0 & ; \text{ if } r \le a \\ S = L-1 & ; \text{ if } r > a \end{array}\right\}$$

ie.

$$\left.\begin{array}{ll} S = 0 & ; \text{ if } r \le 150 \\ S = 256-1 & ; \text{ if } r > 150 \end{array}\right\}$$

$= \underline{255}$

0	0	255	255	0
0	0	~~0~~ 255	0	0
255	0	0	0	0
0	~~0~~ 255	255	0	0
0	0	0	255	0

b] Image Negative Transformation function,

$$S = (L-1) - r$$
$$= (256-1) - r$$
$$= 255 - r$$

I/P grey level (r) → O/P grey level (s)

Resultant Image :-

135	120	40	35	130
120	235	68	205	175
5	140	200	135	210
225	75	55	209	235
195	136	135	0	120

No. of Grey levels $L = 8$ ie. 0 – 7

Q.VII

Grey levels (r_k)	No. of Pixels (n_k)	PDF $P_r(r_k) = n_k/n$	CDF $S_k = \Sigma P_r(r_k)$	$S_k \cdot (L-1)$ $= S_k \cdot (8-1)$	Rounding off
0	0	0	0	0	0
1	50	0.25	0.25	1.75	2
2	0	0	0.25	1.75	2
3	50	0.25	0.50	3.5	4
4	0	0	0.50	3.5	4
5	50	0.25	0.75	5.25	5
6	0	0	0.75	5.25	5
7	50	0.25	1	7	7
	$n = 200$				

Equating Gray levels to the no. of pixels :-

0 → 0
1 → 0
2 → 50
3 → 0
4 → 50
5 → 50
6 → 0
7 → 50

200 Hence verified!