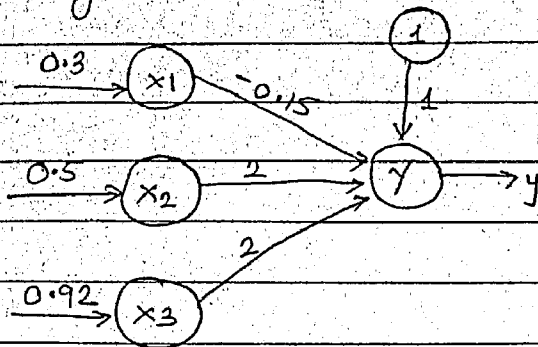


AISC

DATE

Q.I] For the given network calculate net input to the output neuron:-



Soln:-

Here, in this neural network we have 3 input neurons, a bias and an output neurons.

The inputs and at the weights are :-

$$[x_1, x_2, x_3] = [0.3, 0.5, 0.92]$$

$$[w_1, w_2, w_3] = [-0.15, 2, 2]$$

$$\therefore y_{in} = b + \sum_{i=1}^n x_i w_i$$

$$= 1 + (0.3)(-0.15) + (0.5)(2) + (0.92)(2)$$

$$= \underline{\underline{3.795}}$$

Therefore,  $y_{in} = 3.795$  is the net input.

Q.II] For the following fuzzy sets

$$P = \left\{ \frac{0.1}{2} + \frac{0.3}{4} + \frac{0.7}{6} + \frac{0.4}{8} + \frac{0.2}{10} \right\}$$

$$Q = \left\{ \frac{0.1}{0.1} + \frac{0.3}{0.2} + \frac{0.3}{0.3} + \frac{0.4}{0.4} + \frac{0.5}{0.5} + \frac{0.2}{0.6} \right\}$$

$$T = \left\{ \frac{0.1}{0} + \frac{0.7}{0.5} + \frac{0.3}{1} \right\}$$

The following operations are to be performed:-

- 1)  $R = P \times Q$
- 2)  $S = Q \times T$
- 3)  $M = R \circ S$
- 4)  $M = R \bullet S$

Soln:-

1)  $R = P \times Q$

$$R = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.4 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.4 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

$$\mu_R(x_1, y_1) = \min\{\mu_P(x_1), \mu_Q(y_1)\} = \min\{0.1, 0.1\} = 0.1$$

$$\mu_R(x_1, y_2) = \min\{\mu_P(x_1), \mu_Q(y_2)\} = \min\{0.1, 0.3\} = 0.1$$

$$\mu_R(x_1, y_3) = \min\{\mu_P(x_1), \mu_Q(y_3)\} = \min\{0.1, 0.3\} = 0.1$$

$$\mu_R(x_1, y_4) = \min\{\mu_P(x_1), \mu_Q(y_4)\} = \min\{0.1, 0.4\} = 0.1$$

$$\mu_R(x_1, y_5) = \min\{\mu_P(x_1), \mu_Q(y_5)\} = \min\{0.1, 0.5\} = 0.1$$

$$\mu_R(x_1, y_6) = \min\{\mu_P(x_1), \mu_Q(y_6)\} = \min\{0.1, 0.2\} = 0.1$$

||y, we can find  $\mu_R(x_2, y_i)$   $\mu_R(x_3, y_i)$   $\mu_R(x_4, y_i)$  &  $\mu_R(x_5, y_i)$ .



2]  $S = Q \times T$

$$S = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.3 \\ 0.1 & 0.5 & 0.3 \\ 0.1 & 0.2 & 0.2 \end{bmatrix}$$

$$\mu_S(y_1, z_1) = \min\{\mu_Q(y_1), \mu_T(z_1)\} \\ = \min\{0.1, 0.1\} = 0.1$$

$$\mu_S(y_1, z_2) = \min\{\mu_Q(y_1), \mu_T(z_2)\} \\ = \min\{0.1, 0.7\} = 0.1$$

$$\mu_S(y_1, z_3) = \min\{\mu_Q(y_1), \mu_T(z_3)\} \\ = \min\{0.1, 0.3\} = 0.1$$

3]  $M = R \circ S$  [Max-min composition]

	$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	$y_6$		$z_1$	$z_2$	$z_3$
$M = x_1$	0.1	0.1	0.1	0.1	0.1	0.1	$y_1$	0.1	0.1	0.1
$x_2$	0.1	0.3	0.3	0.3	0.3	0.2	$y_2$	0.1	0.3	0.3
$x_3$	0.1	0.3	0.3	0.4	0.5	0.2	$y_3$	0.1	0.3	0.3
$x_4$	0.1	0.3	0.3	0.4	0.4	0.2	$y_4$	0.1	0.4	0.3
$x_5$	0.1	0.2	0.2	0.2	0.2	0.2	$y_5$	0.1	0.5	0.3
							$y_6$	0.1	0.2	0.2

~~$$\therefore \mu_M(x_1, z_1) = \max\{\min(\mu_R(x_1, y_1), \mu_S(y_1, z_1))\} \\ = \max\{\min(0.1, 0.1)\} \\ = \max\{0.1\}$$~~

$$\mu_M(x_1, z_1) = \max\{\min(\mu_R(x_1, y_1), \mu_S(y_1, z_1)), \\ \min(\mu_R(x_1, y_2), \mu_S(y_2, z_1)), \\ \min(\mu_R(x_1, y_3), \mu_S(y_3, z_1)), \\ \min(\mu_R(x_1, y_4), \mu_S(y_4, z_1)), \\ \min(\mu_R(x_1, y_5), \mu_S(y_5, z_1)), \\ \min(\mu_R(x_1, y_6), \mu_S(y_6, z_1))\}$$

$$\begin{aligned} \therefore \mu_M(x_1, z_1) &= \max\{\min(0.1, 0.1), \min(0.1, 0.1), \min(0.1, 0.1), \\ &\quad \min(0.1, 0.1), \min(0.1, 0.1), \min(0.1, 0.1)\} \\ &= \max\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1\} = \underline{0.1} \end{aligned}$$

$$\begin{aligned} \therefore \mu_M(x_1, z_2) &= \max\{\min(0.1, 0.1), \min(0.1, 0.3), \min(0.1, 0.3), \\ &\quad \min(0.1, 0.4), \min(0.1, 0.5), \min(0.1, 0.2)\} \\ &= \max\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1\} = \underline{0.1} \end{aligned}$$

$$\begin{aligned} \mu_M(x_1, z_3) &= \max\{\min(0.1, 0.1), \min(0.1, 0.3), \min(0.1, 0.3), \\ &\quad \min(0.1, 0.3), \min(0.1, 0.3), \min(0.1, 0.2)\} \\ &= \max\{0.1, 0.1, 0.1, 0.1, 0.1, 0.1\} = \underline{0.1} \end{aligned}$$

	$z_1$	$z_2$	$z_3$
$\therefore M = x_1$	0.1	0.1	0.1
$x_2$	0.1	0.3	0.3
$x_3$	0.1	0.5	0.3
$x_4$	0.1	0.4	0.3
$x_5$	0.1	0.2	0.2
$x_6$	0.1		

4]  $M = R \circ S$

$$\begin{aligned} \mu_M(x_1, z_1) &= \max\{\mu_R(x_1, y_1) \cdot \mu_S(y_1, z_1), \\ &\quad \mu_R(x_1, y_2) \cdot \mu_S(y_2, z_1), \\ &\quad \mu_R(x_1, y_3) \cdot \mu_S(y_3, z_1), \\ &\quad \mu_R(x_1, y_4) \cdot \mu_S(y_4, z_1), \\ &\quad \mu_R(x_1, y_5) \cdot \mu_S(y_5, z_1), \\ &\quad \mu_R(x_1, y_6) \cdot \mu_S(y_6, z_1)\} \end{aligned}$$



$$\therefore \mu_M(x_1, x_1) = \max\{0.01, 0.01, 0.01, 0.01, 0.01, 0.01\}$$

$$= 0.01$$

$$\mu_M(x_1, x_2) = \max\{0.01, 0.03, 0.03, 0.04, 0.05, 0.02\}$$

$$= 0.05$$

$$\mu_M(x_1, x_3) = \max\{0.01, 0.03, 0.03, 0.03, 0.03, 0.02\}$$

$$= 0.03$$

$$\therefore M = R \circ S$$

	$x_1$	$x_2$	$x_3$
$x_1$	0.01	0.05	0.03
$x_2$	0.03	0.15	0.09
$x_3$	0.05	0.25	0.15
$x_4$	0.04	0.2	0.12
$x_5$	0.02	0.1	0.06

Q. III]  $A = \left\{ \begin{array}{c} 0.2 + 0.3 + 0.4 + 0.7 + 0.1 \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \end{array} \right\}$

$$B = \left\{ \begin{array}{c} 0.4 + 0.5 + 0.6 + 0.8 + 0.9 \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \end{array} \right\}$$

$\lambda = 0.6$  for the foll operations :-  
 $\underline{\underline{A \cup B}}, \underline{\underline{A \cap B}}$

Soln  $\underline{\underline{A \cup B}} = \max\{\mu_A(x), \mu_B(x)\}$

$$= \left\{ \begin{array}{c} 0.4 + 0.5 + 0.6 + 0.8 + 0.9 \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \end{array} \right\}$$

$$\underline{\underline{A \cap B}} = 1 - \underline{\underline{A \cup B}}$$

$$= \left\{ \begin{array}{c} 0.6 + 0.5 + 0.4 + 0.2 + 0.1 \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \end{array} \right\}$$

$$\therefore \underline{\underline{(A \cup B)}}_{0.6} = \left\{ \begin{array}{c} 1 + 0 + 0 + 0 + 0 \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \end{array} \right\} \text{ OR } \{x_1\}$$

$$\therefore \underline{A \cap B} = \min \{ \mu_A(x), \mu_B(x) \}$$

$$\underline{\tilde{A} \cap \tilde{B}} = \left\{ \frac{0.2}{x_1} + \frac{0.3}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.1}{x_5} \right\}$$

$$\therefore (\underline{\tilde{A} \cap \tilde{B}}) = 1 - \underline{A \cap B}$$

$$= \left\{ \frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5} \right\}$$

$$\therefore (\underline{\tilde{A} \cap \tilde{B}})_{0.6} = \left\{ \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{0}{x_4} + \frac{1}{x_5} \right\}$$

$$\text{OR } \{ x_1, x_2, x_3, x_5 \}$$

Q. II

$$\underline{A} = \left\{ \frac{0.3}{x_1} + \frac{0.4}{x_2} + \frac{0.5}{x_3} + \frac{0.7}{x_4} + \frac{1}{x_5} \right\}$$

$$\underline{B} = \left\{ \frac{0.1}{x_1} + \frac{0.8}{x_2} + \frac{0.7}{x_3} + \frac{1}{x_4} + \frac{0.6}{x_5} \right\}$$

Determine the following:-

1)  $\underline{A \cap B}$  2)  $\underline{A \cup B}$  3)  $\underline{\overline{A \cup B}}$  4)  $\underline{\overline{A \cap B}}$  5)  $\underline{A \cap \overline{A}}$

1)  $\underline{A \cap B} = \min \{ \mu_A(x), \mu_B(x) \}$

$$= \left\{ \frac{0.3}{x_1} + \frac{0.4}{x_2} + \frac{0.5}{x_3} + \frac{0.7}{x_4} + \frac{0.6}{x_5} \right\}$$

2)  $\underline{A \cup B} = \max \{ \mu_A(x), \mu_B(x) \}$

$$= \left\{ \frac{0.7}{x_1} + \frac{0.8}{x_2} + \frac{0.7}{x_3} + \frac{1}{x_4} + \frac{1}{x_5} \right\}$$

3)  $\underline{\overline{A \cup B}} = 1 - \underline{A \cup B}$

$$= \left\{ \frac{0.3}{x_1} + \frac{0.2}{x_2} + \frac{0.3}{x_3} + \frac{0}{x_4} + \frac{0}{x_5} \right\}$$

4)  $\underline{\overline{A \cap B}} = \max \{ \mu_{\overline{A}}(x), \mu_{\overline{B}}(x) \}$

$$\therefore \underline{\overline{A}} = 1 - \underline{A} = \left\{ \frac{0.7}{x_1} + \frac{0.6}{x_2} + \frac{0.5}{x_3} + \frac{0.3}{x_4} + \frac{0}{x_5} \right\}$$



$$\bar{B} = 1 - B$$

$$= \left\{ \frac{0.8}{x_1} + \frac{0.2}{x_2} + \frac{0.8}{x_3} + \frac{0}{x_4} + \frac{0.4}{x_5} \right\}$$

$$\therefore \bar{A} \cup \bar{B} = \left\{ \frac{0.7}{x_1} + \frac{0.6}{x_2} + \frac{0.5}{x_3} + \frac{0.3}{x_4} + \frac{0.4}{x_5} \right\}$$

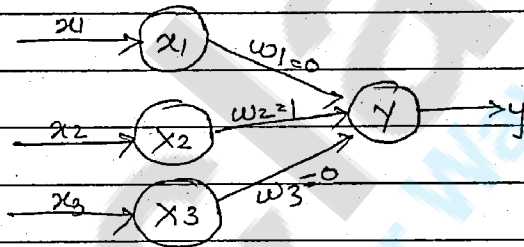
5)  $\underline{A} \cap \bar{B} = \min\{\mu_A(x), \mu_{\bar{B}}(x)\}$

$$= \left\{ \frac{0.3}{x_1} + \frac{0.4}{x_2} + \frac{0.5}{x_3} + \frac{0.3}{x_4} + \frac{0.7}{x_5} \right\}$$

Q. 5]

Truth Table

$x_1$	$x_2$	$x_3$	$t$
2	1	-1	-1
0	1	-1	1



learning rate ( $\alpha$ ) = 1 & let  $\theta = 0$  &  $b = 0$   
 Initial weights :  $w_1 = 0, w_2 = 1, w_3 = 0$   
 with bipolar activation function as;

$$f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } y_{in} = \theta \\ -1 & \text{if } y_{in} \leq \theta \end{cases}$$

EPOCH 1:

Inputs	Target	Net Input	Calculated/Op ( $\gamma$ )	Weight Changes			Weights			
				$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	$w_1$	$w_2$	$w_3$	$b$
$x_1, x_2, x_3$	( $t$ )	( $y_{in}$ )	( $\gamma$ )				(0)	(1)	(0)	(0)
2, 1, -1	-1	1	1	-2	-1	1	-2	0	1	-1
0, 1, -1	1	-2	-1	0	1	-1	-2	1	0	0

$$\therefore y_{in} = \sum x_i w_i + b$$

$$= x_1 w_1 + x_2 w_2 + x_3 w_3 + 0$$

$$= (2)(0) + (1)(1) + (-1)(0) + 0$$

$$= \underline{1}$$

Since  $t \neq y$  we need to calculate the updated weights as follows:

$$\begin{aligned}w_1(\text{new}) &= w_1(\text{old}) + \alpha t_1 x_1 \\ &= 0 + (1)(-1)(2) \\ &= -2\end{aligned}$$

$$\begin{aligned}b(\text{new}) &= b(\text{old}) + \alpha t_1 \\ &= 0 + (1)(-1) \\ &= -1\end{aligned}$$

$$\begin{aligned}w_2(\text{new}) &= w_2(\text{old}) + \alpha t_1 x_2 \\ &= 1 + (1)(-1)(1) \\ &= 0\end{aligned}$$

$$\begin{aligned}w_3(\text{new}) &= w_3(\text{old}) + \alpha t_1 x_3 \\ &= 0 + (1)(-1)(-1) \\ &= 1\end{aligned}$$

Now again we calculate  $y_{\text{in}} = \sum x_i w_i + b$

$$\begin{aligned}y_{\text{in}} &= x_1 w_1 + x_2 w_2 + x_3 w_3 + b \\ &= (0)(-2) + (1)(0) + (-1)(1) + (-1) \\ &= -1 - 1 = -2\end{aligned}$$

Since  $t \neq y$  we need to calculate the updated weights :-

$$\begin{aligned}w_1(\text{new}) &= w_1(\text{old}) + \alpha t_2 x_1 \\ &= -2 + (1)(1)(0) \\ &= -2\end{aligned}$$

$$\begin{aligned}w_2(\text{new}) &= w_2(\text{old}) + \alpha t_2 x_2 \\ &= 0 + (1)(1)(1) \\ &= 1\end{aligned}$$

$$\begin{aligned}w_3(\text{new}) &= w_3(\text{old}) + \alpha t_2 x_3 \\ &= 1 + (1)(1)(-1) \\ &= 0\end{aligned}$$

$$\begin{aligned}b(\text{new}) &= b(\text{old}) + \alpha t_2 \\ &= (-1) + (1)(1) \\ &= 0\end{aligned}$$



Inputs	Target	Net Input	Calculated o/p	Weight Changes			
				$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	b
$x_1, x_2, x_3$	(t)	$y_{in}$	y	$w_1$	$w_2$	$w_3$	b
2, 1, -1	-1	-3	-1	0	0	0	-2, 1, 0, 0
0, 1, -1	1	1	1	0	0	0	-2, 1, 0, 0

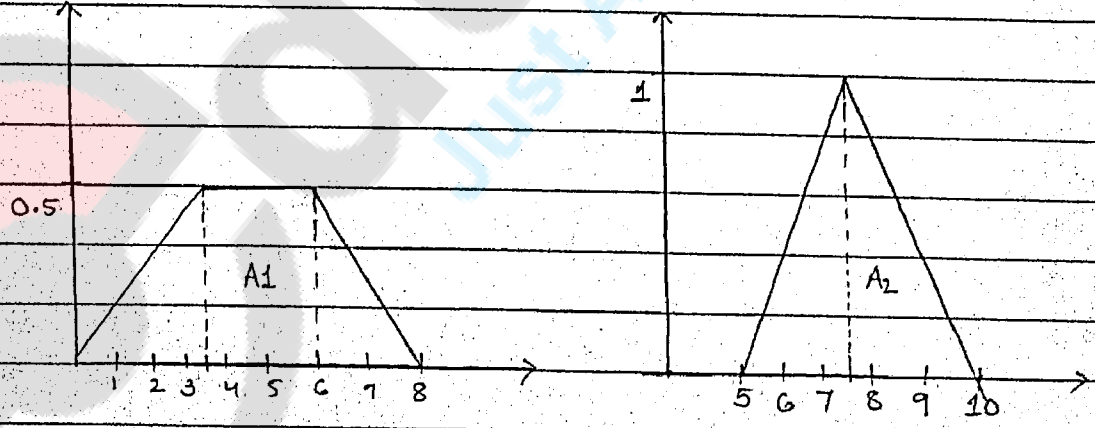
$$\begin{aligned} \therefore y_{in} &= w_1 x_1 + w_2 x_2 + w_3 x_3 + b \\ &= (-2)(2) + (1)(1) + (0)(-1) + 0 \\ &= -4 + 1 + 0 + 0 \\ &= -3 \end{aligned}$$

Now  
Since  $t = y$  we do not update the weights.

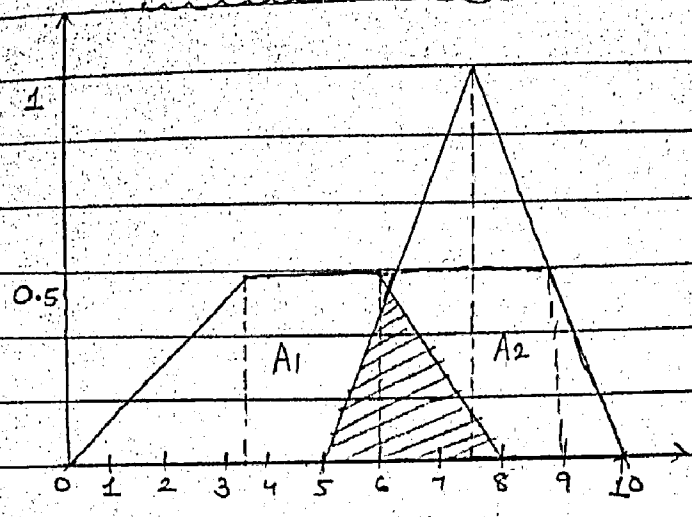
$$\begin{aligned} \therefore y_{in} &= w_1 x_1 + w_2 x_2 + w_3 x_3 + b \\ &= (-2)(0) + (1)(1) + (0)(-1) + 0 \\ &= 0 + 1 + 0 + 0 \\ &= 1 \end{aligned}$$

Now since  $t = y$  we do not update the weight.  
Also since all the target values (t) is equal to the calculated output (y) we stop after epoch 2.

Q. VI



### Combined fuzzy Curve



1) Centroid Method :-

$$x^* = \frac{\sum A_i \bar{x}_i}{\sum A_i}$$

∴ Area I = Area of Trapezium

$$= \frac{1}{2}((10-0) + (9-3.5)) \times 0.5$$

$$= \frac{1}{2}(10 + 5.5) \times 0.5$$

$$= \underline{3.875}$$

∴ Area II = Area of Triangle

Centre of sum =  $\frac{1}{2}(9-6) \times 0.5$

$$= \underline{0.75}$$

$$\therefore x^* = \frac{(3.875) \left(\frac{22.5}{4}\right) + (0.75) \left(\frac{22.5}{8}\right)}{(3.875) + (0.75)}$$

$$= \frac{27.421875}{4.625} = \underline{5.929}$$



2) Centre of Mass :- (Intersecting Areas are added twice)

$$x^* = \frac{\sum A_i x_c}{\sum A_i}$$

where,  $A_i$  is the area of the known geometric figures.  
 $x_c$  is the center of the known geometric figures.

$$\begin{aligned} \therefore \text{Area of } A_1 &= \frac{1}{2}((8-0) + (6-3.5)) \times 0.5 \\ &= \frac{1}{2}(8 + 2.5) \times 0.5 \\ &= \frac{1}{2}(10.5) \times 0.5 \\ &= 2.625 \end{aligned}$$

$$\begin{aligned} \text{Area of } A_2 &= \frac{1}{2}(10-5) \times 1 \\ &= \frac{1}{2}(5) \times 1 \\ &= 2.5 \end{aligned}$$

$$\begin{aligned} \therefore x^* &= \frac{A_1 x_{c1} + A_2 x_{c2}}{A_1 + A_2} \\ &= \frac{(2.625)(4) + (2.5)(7.5)}{2.625 + 2.5} \\ &= 5.707 \end{aligned}$$

3) Weighted Average :-

$$\begin{aligned} x^* &= \frac{\sum H(x) x_c}{\sum H(x)} \\ &= \frac{(4)(0.5) + (7.5)(1)}{(0.5 + 1)} \\ &= 6.33 \end{aligned}$$

4) Centre of Largest Area :- (Mean value of fuzzy set having largest area)

Since Area of  $A_1 >$  Area of  $A_2$ ,

$$x^* = \frac{3.5 + 6}{2} = \frac{9.5}{2} = 4.75 \quad x^* = \frac{3.5 + 4 + 5 + 6}{4} = \underline{4.625}$$

5) Mean Max Method :-

$$x^* = \underline{7.5}$$

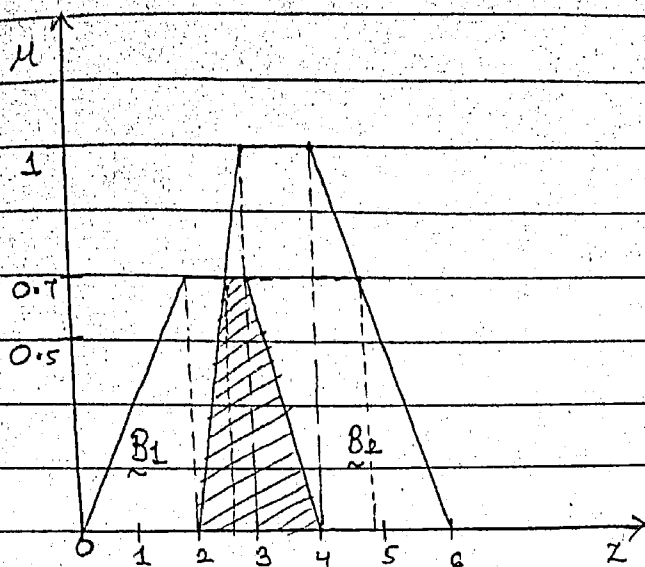
6) First of Maxima :-

$$x^* = \underline{7.5}$$

7) Last of Maxima :-

$$x^* = \underline{7.5}$$





1) Centroid Method:-

$$x^* = \frac{\sum A_i \bar{x}_i}{\sum A_i}$$

$$\begin{aligned} \text{Area I} &= \frac{1}{2} ((6-0) + (4.8-2)) \times 0.7 \\ &= \frac{1}{2} (6 + 2.8) \times 0.7 \\ &= \underline{3.08} \end{aligned}$$

$$\begin{aligned} \text{Area II} &= \frac{1}{2} ((4.8-2.5) + (4-3)) \times 0.3 \\ &= \frac{1}{2} (2.3 + 1) \times 0.3 \\ &= \underline{0.495} \end{aligned}$$

$$\begin{aligned} \therefore x^* &= \frac{A_1 \bar{x}_1 + A_2 \bar{x}_2}{A_1 + A_2} \\ &= \frac{(1.75)(3.08)(3.2) + (0.495)(8.575)}{3.08 + 0.495} \\ &= \underline{\underline{3.25}} \end{aligned}$$

2) Centre of Sums:-  $x^* = \frac{\sum A_i x_c}{\sum A_i}$

$$\begin{aligned} \text{Area of } B_1 &= \frac{1}{2}((4-0) + (3-2)) \times 0.7 \\ &= \frac{1}{2}(4+1) \times 0.7 \\ &= \underline{1.75} \end{aligned}$$

$$\begin{aligned} \text{Area of } B_2 &= \frac{1}{2}((6-2) + (4-3)) \times 1 \\ &= \frac{1}{2}(4+1) \times 1 \\ &= \underline{2.5} \end{aligned}$$

$$\begin{aligned} \therefore x^* &= \frac{A_1 x_{c1} + A_2 x_{c2}}{A_1 + A_2} \\ &= \frac{(1.75)(2) + (2.5)(4)}{1.75 + 2.5} \\ &= \underline{3.17} \end{aligned}$$

3) Centre of Largest Area:-

From method 2 we know that Area of  $B_2$  is largest.

$$\therefore x^* = \frac{3+4}{2} = \frac{7}{2} = \underline{3.5}$$

4) Weighted Average Method:-

$$\begin{aligned} x^* &= \frac{\sum M(x) x}{\sum M(x)} \\ &= \frac{(0.7)(2) + (1)(4)}{0.7+1} \\ &= \underline{3.17} \end{aligned}$$



5) Mean Max Method:-

$$x^* = \frac{a+b}{2}$$

$$= \frac{3+4}{2} = \underline{3.5}$$

6) First of Maxima:-

$$x^* = 3$$

7) Last of Maxima:-

$$x^* = 4$$