

B+ Trees

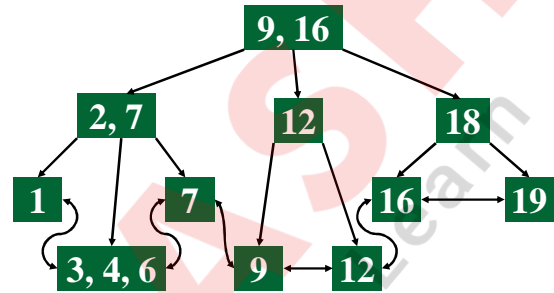
- Similar to B trees, with a few slight differences
- All data is stored at the leaf nodes (*leaf pages*); all other nodes (*index pages*) only store keys
- Leaf pages are linked to each other
- Keys may be duplicated; every key to the right of a particular key is \geq to that key

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B+ Tree Example



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B+ Tree Insertion

- Insert at bottom level
- If leaf page overflows, split page and copy middle element to next index page
- If index page overflows, split page and move middle element to next index page

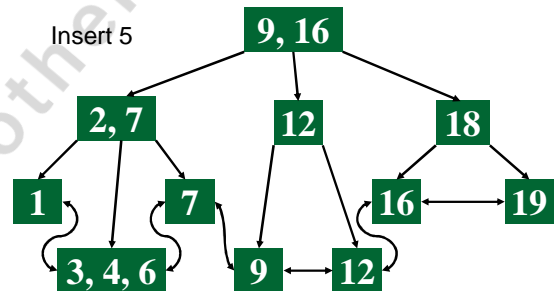
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B+ Tree Insertion Example

Insert 5



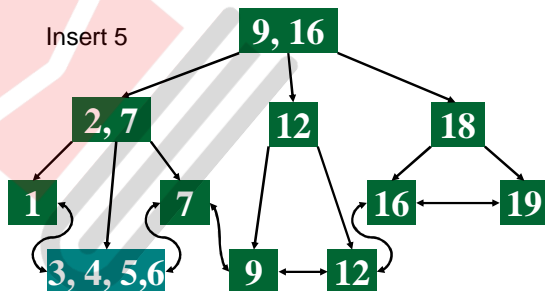
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B+ Tree Insertion Example

Insert 5



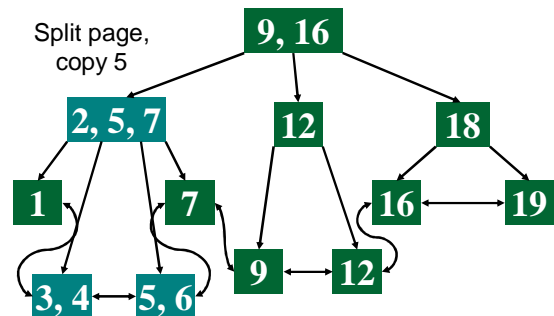
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B+ Tree Insertion Example

Split page,
copy 5



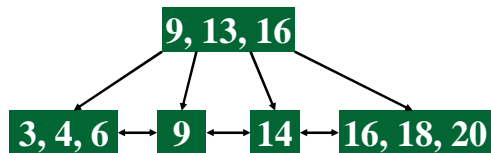
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B+ Tree Insertion Example 2

Insert 17



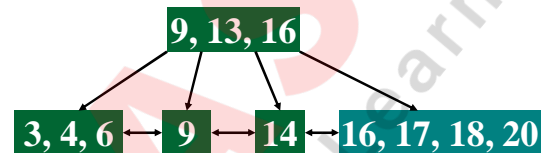
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B+ Tree Insertion Example 2

Insert 17



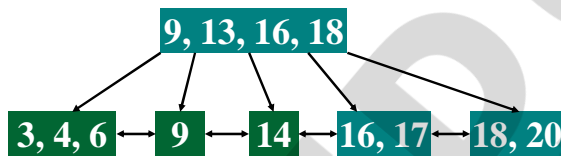
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B+ Tree Insertion Example 2

Split leaf
page, copy 18



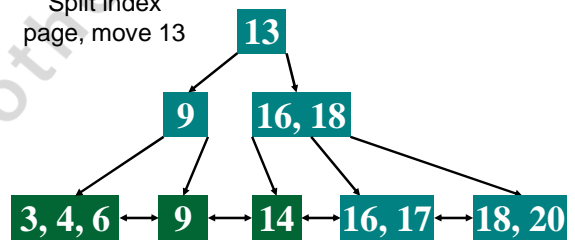
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B+ Tree Insertion Example 2

Split index
page, move 13



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B+ Tree Deletion

- Delete key and data from leaf page
- If leaf page underflows, merge with sibling and delete key in between them
- If index page underflows, merge with sibling and move down key in between them

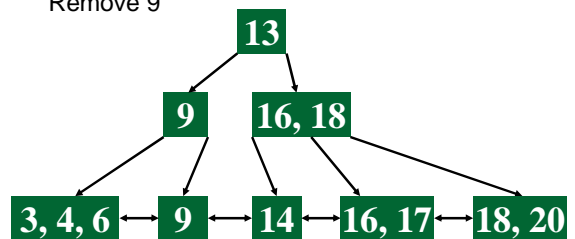
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B+ Tree Deletion Example

Remove 9



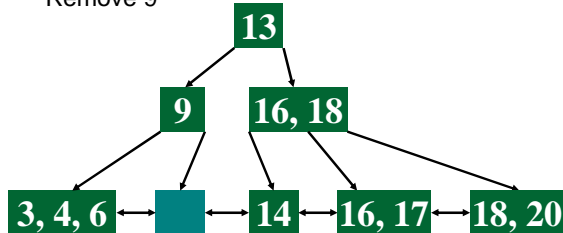
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B+ Tree Deletion Example

Remove 9



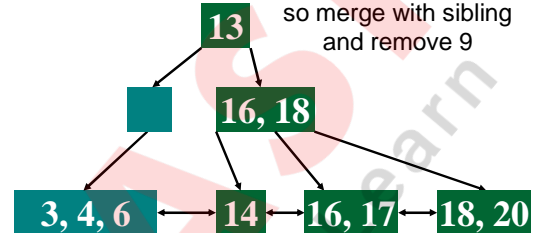
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B+ Tree Deletion Example

Leaf page underflow,
so merge with sibling
and remove 9



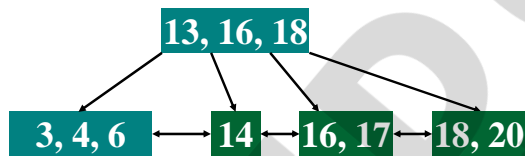
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B+ Tree Deletion Example

Index page underflow,
so merge with sibling
and demote 13



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Threaded Trees

- Binary trees have a lot of wasted space: the leaf nodes each have 2 null pointers
- We can use these pointers to help us in inorder traversals
- We have the pointers reference the next node in an inorder traversal; called *threads*
- We need to know if a pointer is an actual link or a thread, so we keep a boolean for each pointer

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Threaded Tree Code

Example code:

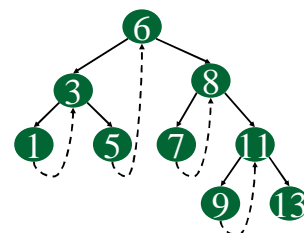
```
class Node {
    Node left, right;
    boolean leftThread, rightThread;
}
```

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Threaded Tree Example



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Threaded Tree Traversal

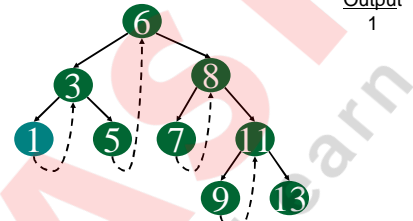
- We start at the leftmost node in the tree, print it, and follow its right thread
- If we follow a thread to the right, we output the node and continue to its right
- If we follow a link to the right, we go to the leftmost node, print it, and continue

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Threaded Tree Traversal



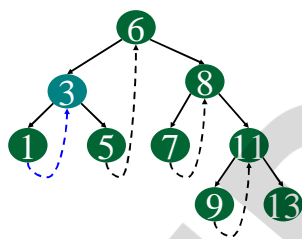
Start at leftmost node, print it

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Threaded Tree Traversal



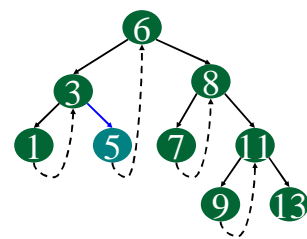
Follow thread to right, print node

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Threaded Tree Traversal



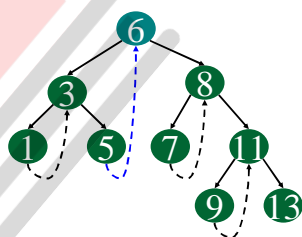
Follow link to right, go to leftmost node and print

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Threaded Tree Traversal



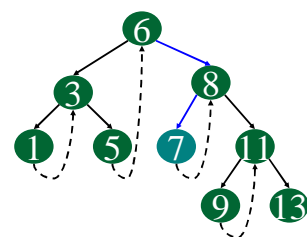
Follow thread to right, print node

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Threaded Tree Traversal



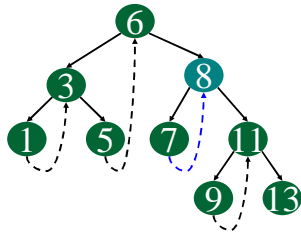
Follow link to right, go to leftmost node and print

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Threaded Tree Traversal



Output
1
3
5
6
7
8

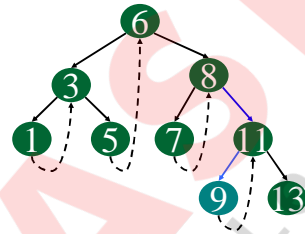
Follow thread to right, print node

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Threaded Tree Traversal



Output
1
3
5
6
7
8
9

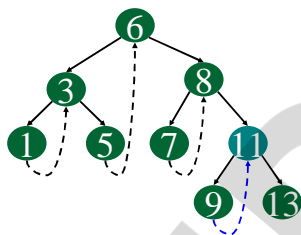
Follow link to right, go to
leftmost node and print

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Threaded Tree Traversal



Output
1
3
5
6
7
8
9
11

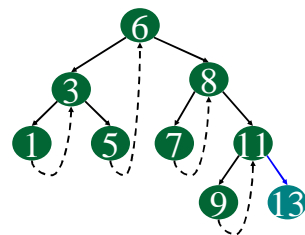
Follow thread to right, print node

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Threaded Tree Traversal



Output
1
3
5
6
7
8
9
11
13

Follow link to right, go to
leftmost node and print

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Threaded Tree Traversal Code

```
Node leftMost(Node n) {
    Node ans = n;
    if (ans == null) {
        return null;
    }
    while (ans.left != null) {
        ans = ans.left;
    }
    return ans;
}

void inOrder(Node n) {
    Node cur = leftMost(n);
    while (cur != null) {
        print(cur);
        if (cur.rightThread) {
            cur = cur.right;
        } else {
            cur = leftMost(cur.right);
        }
    }
}
```

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Threaded Tree Modification

- We're still wasting pointers, since half of our leaves' pointers are still null
- We can add threads to the previous node in an inorder traversal as well, which we can use to traverse the tree backwards or even to do postorder traversals

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Threaded Tree Modification

