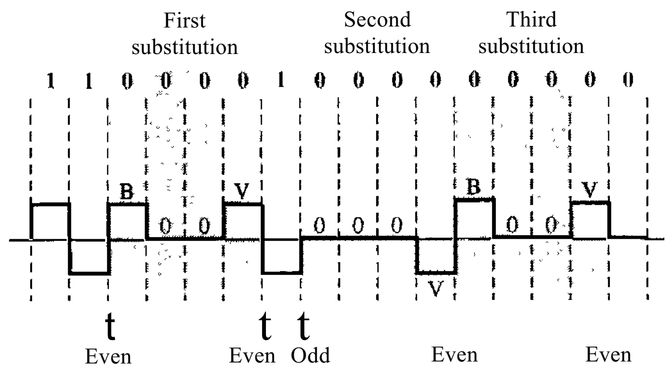
maintain the even number of nonzero pulses after each substitution. The two rules can be stated as follows:

1. If the number of nonzero pulses after the last substitution is odd, the substitution pattern will be OOOV, which makes the total number of nonzero pulses even.
2. If the number of nonzero pulses after the last substitution is even, the substitution pattern will be BOOV, which makes the total number of nonzero pulses even.

Figure 4.20 shows an example.

Figure 4.20 *Different situations in HDB3 scrambling technique*



There are several points we need to mention here. First, before the first substitution, the number of nonzero pulses is even, so the first substitution is BODY. After this substitution, the polarity of the 1 bit is changed because the AMI scheme, after each substitution, must follow its own rule. After this bit, we need another substitution, which is OOOV because we have only one nonzero pulse (odd) after the last substitution. The third substitution is BOOV because there are no nonzero pulses after the second substitution (even).

HDB3 substitutes four consecutive zeros with OOOV or BOOV depending on the number of nonzero pulses after the last substitution.

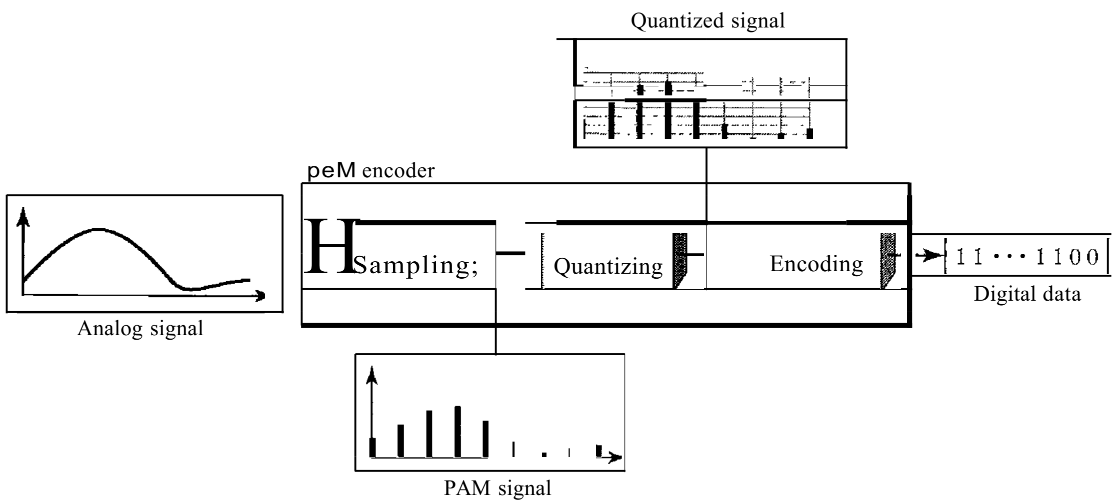
4.2 ANALOG-TO-DIGITAL CONVERSION

The techniques described in Section 4.1 convert digital data to digital signals. Sometimes, however, we have an analog signal such as one created by a microphone or camera. We have seen in Chapter 3 that a digital signal is superior to an analog signal. The tendency today is to change an analog signal to digital data. In this section we describe two techniques, pulse code modulation and delta modulation. After the digital data are created (digitization), we can use one of the techniques described in Section 4.1 to convert the digital data to a digital signal.

Pulse Code Modulation (PCM)

The most common technique to change an analog signal to digital data (digitization) is called pulse code modulation (PCM). A PCM encoder has three processes, as shown in Figure 4.21.

Figure 4.21 *Components of PCM encoder*



1. The analog signal is sampled.
2. The sampled signal is quantized.
3. The quantized values are encoded as streams of bits.

Sampling

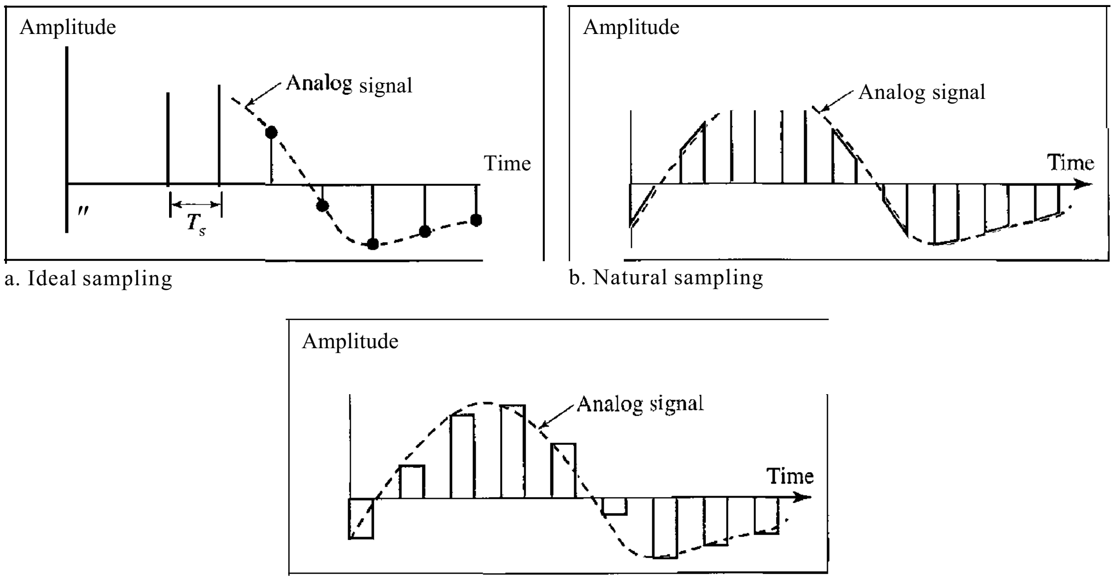
The first step in PCM is sampling. The analog signal is sampled every T_s s, where T_s is the sample interval or period. The inverse of the sampling interval is called the sampling rate or sampling frequency and denoted by f_s, where $f_s = 1/T_s$. There are three sampling methods-ideal, natural, and flat-top-as shown in Figure 4.22.

In ideal sampling, pulses from the analog signal are sampled. This is an ideal sampling method and cannot be easily implemented. In natural sampling, a high-speed switch is turned on for only the small period of time when the sampling occurs. The result is a sequence of samples that retains the shape of the analog signal. The most common sampling method, called sample and hold, however, creates flat-top samples by using a circuit.

The sampling process is sometimes referred to as pulse amplitude modulation (PAM). We need to remember, however, that the result is still an analog signal with nonintegral values.

Sampling Rate One important consideration is the sampling rate or frequency. What are the restrictions on T_s? This question was elegantly answered by Nyquist. According to the Nyquist theorem, to reproduce the original analog signal, one necessary condition is that the sampling rate be at least twice the highest frequency in the original signal.

Figure 4.22 *Three different sampling methods for PCM*



a. Ideal sampling

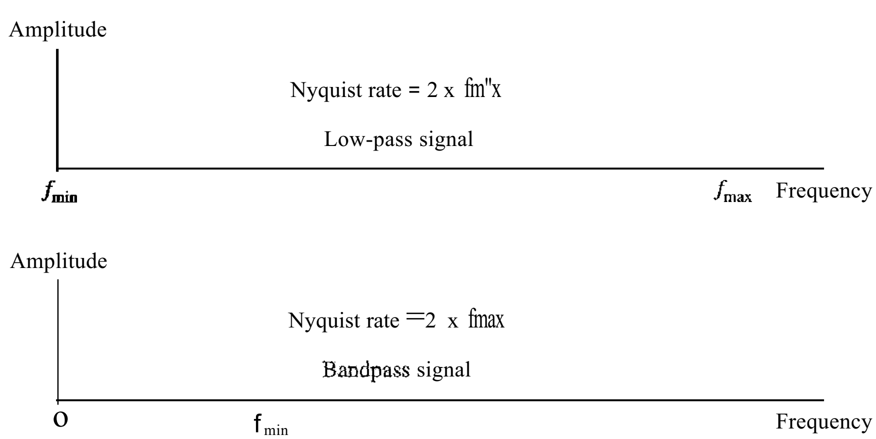
b. Natural sampling

c. Flat-top sampling

According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

We need to elaborate on the theorem at this point. First, we can sample a signal only if the signal is band-limited. In other words, a signal with an infinite bandwidth cannot be sampled. Second, the sampling rate must be at least 2 times the highest frequency, not the bandwidth. If the analog signal is low-pass, the bandwidth and the highest frequency are the same value. If the analog signal is bandpass, the bandwidth value is lower than the value of the maximum frequency. Figure 4.23 shows the value of the sampling rate for two types of signals.

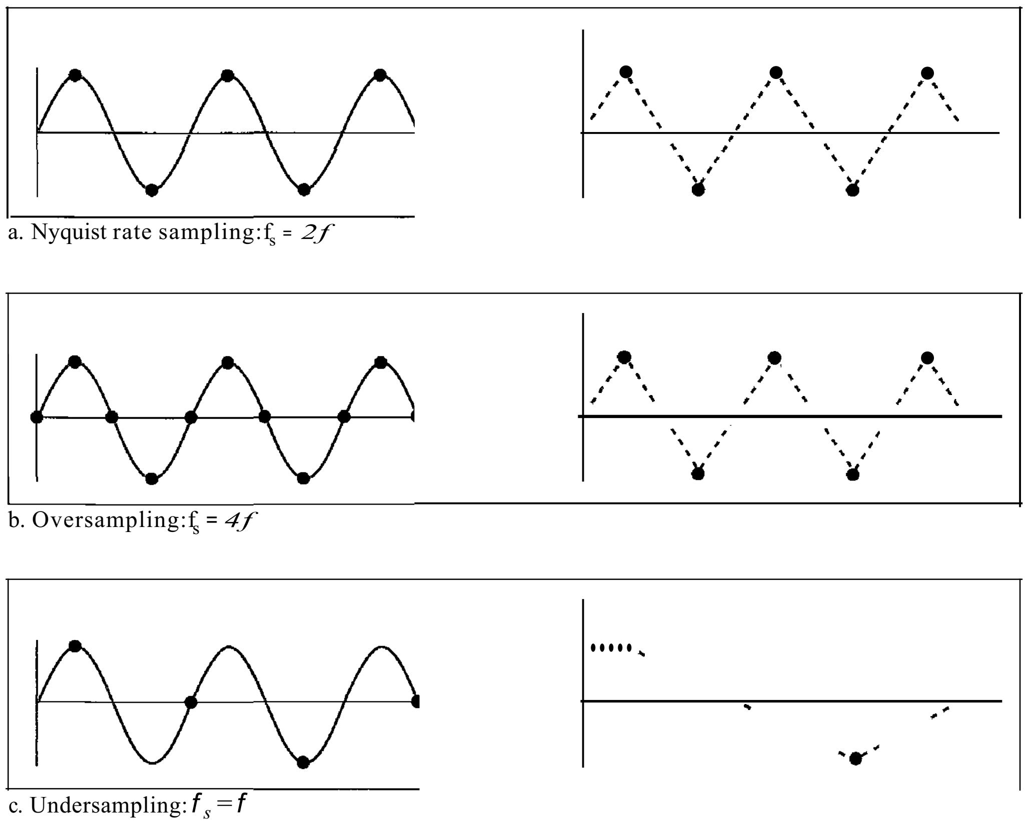
Figure 4.23 *Nyquist sampling rate for low-pass and bandpass signals*



Example 4.6

For an intuitive example of the Nyquist theorem, let us sample a simple sine wave at three sampling rates: $fs = 4f$ (2 times the Nyquist rate)'/s $= 2f$ (Nyquist rate), and $f_s = f$ (one-half the Nyquist rate). Figure 4.24 shows the sampling and the subsequent recovery of the signal.

Figure 4.24 *Recovery of a sampled sine wave for different sampling rates*

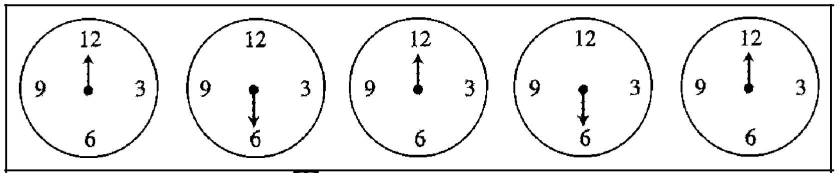


It can be seen that sampling at the Nyquist rate can create a good approximation of the original sine wave (part a). Oversampling in part b can also create the same approximation, but it is redundant and unnecessary. Sampling below the Nyquist rate (part c) does not produce a signal that looks like the original sine wave.

Example 4.7

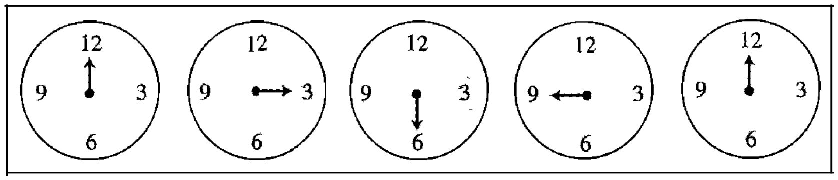
As an interesting example, let us see what happens if we sample a periodic event such as the revolution of a hand of a clock. The second hand of a clock has a period of 60 s. According to the Nyquist theorem, we need to sample the hand (take and send a picture) every 30 s ($T_s = \frac{1}{2}T$ or $f_s = 2f$). In Figure 4.25a, the sample points, in order, are 12, 6, 12, 6, 12, and 6. The receiver of the samples cannot tell if the clock is moving forward or backward. In part b, we sample at double the Nyquist rate (every 15 s). The sample points, in order, are 12, 3, 6, 9, and 12. The clock is moving forward. In part c, we sample below the Nyquist rate ($T_s = \frac{3}{4}T$ or $f_s = \frac{4}{3}f$). The sample points, in order, are 12, 9, 6, 3, and 12. Although the clock is moving forward, the receiver thinks that the clock is moving backward.

Figure 4.25 *Sampling ofa clock with only one hand*



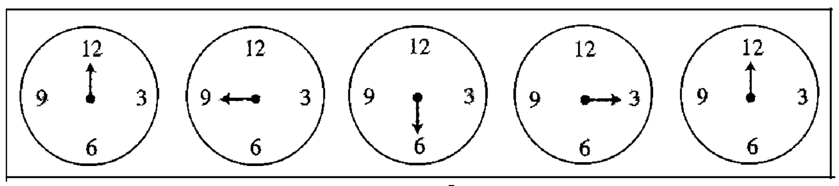
Samples can mean that the clock is moving either forward or backward. (12-6-12-6-12)

a. Sampling at Nyquist rate: $T_s = \frac{1}{2}T$



Samples show clock is moving forward. (12·3-6-9-12)

b. Oversampling (above Nyquist rate): $T_s = \frac{1}{4}T$



Samples show clock is moving backward. (12-9-6-3-12)

c. Undersampling (below Nyquist rate): $T_s = \frac{3}{4}T$

Example 4.8

An example related to Example 4.7 is the seemingly backward rotation of the wheels of a forward-moving car in a movie. This can be explained by undersampling. A movie is filmed at 24 frames per second. If a wheel is rotating more than 12 times per second, the undersampling creates the impression of a backward rotation.

Example 4.9

Telephone companies digitize voice by assuming a maximum frequency of 4000 Hz. The sampling rate therefore is 8000 samples per second.

Example 4.10

A complex low-pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution

The bandwidth of a low-pass signal is between 0 andj, where f is the maximum frequency in the signal. Therefore, we can sample this signal at 2 times the highest frequency (200 kHz). The sampling rate is therefore 400,000 samples per second.

Example 4.1J

A complex bandpass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution

We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. We do not know the maximum frequency in the signal.

Quantization

The result of sampling is a series of pulses with amplitude values between the maximum and minimum amplitudes of the signal. The set of amplitudes can be infinite with nonintegral values between the two limits. These values cannot be used in the encoding process. The following are the steps in quantization:

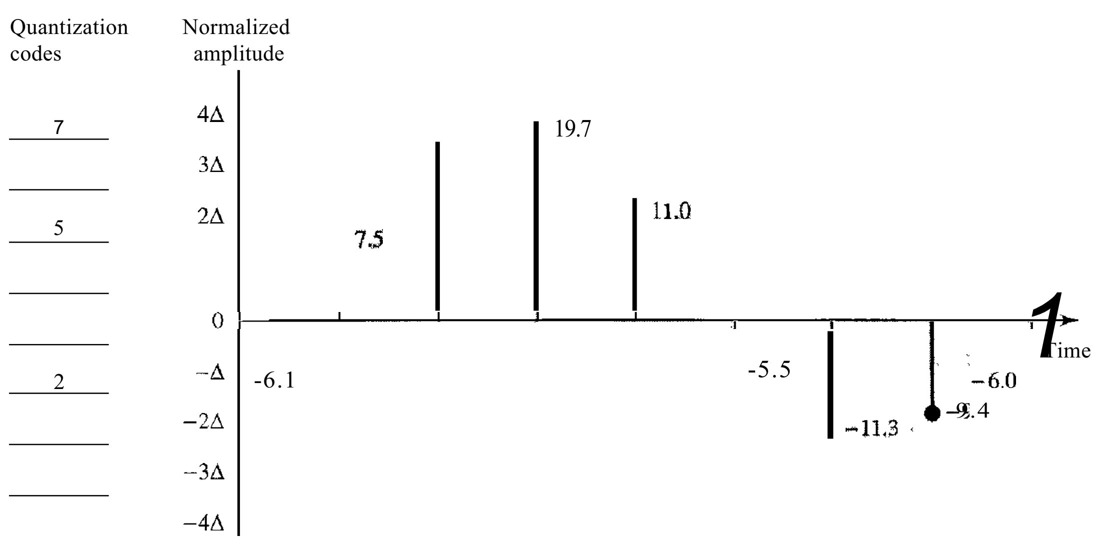
1. We assume that the original analog signal has instantaneous amplitudes between V_{min} and V_{max}.
2. We divide the range into L zones, each of height Δ (delta).

$$\Delta = \frac{V_{max} - V_{min}}{L}$$

3. We assign quantized values of 0 to $L - 1$ to the midpoint of each zone.
4. We approximate the value of the sample amplitude to the quantized values.

As a simple example, assume that we have a sampled signal and the sample amplitudes are between -20 and +20 V. We decide to have eight levels ($L = 8$). This means that $\Delta = 5$ V. Figure 4.26 shows this example.

Figure 4.26 *Quantization and encoding of a sampled signal*



Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	O	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2		2	2
Encoded words	010	101	111	111	110	010	001	010	OW

We have shown only nine samples using ideal sampling (for simplicity). The value at the top of each sample in the graph shows the actual amplitude. In the chart, the first row is the normalized value for each sample (actual amplitude/Δ). The quantization process selects the quantization value from the middle of each zone. This means that the normalized quantized values (second row) are different from the normalized amplitudes. The difference is called the *normalized error* (third row). The fourth row is the quantization code for each sample based on the quantization levels at the left of the graph. The encoded words (fifth row) are the final products of the conversion.

Quantization Levels In the previous example, we showed eight quantization levels. The choice of L, the number of levels, depends on the range of the amplitudes of the analog signal and how accurately we need to recover the signal. If the amplitude of a signal fluctuates between two values only, we need only two levels; if the signal, like voice, has many amplitude values, we need more quantization levels. In audio digitizing, L is normally chosen to be 256; in video it is normally thousands. Choosing lower values of L increases the quantization error if there is a lot of fluctuation in the signal.

Quantization Error One important issue is the error created in the quantization process. (Later, we will see how this affects high-speed modems.) Quantization is an approximation process. The input values to the quantizer are the real values; the output values are the approximated values. The output values are chosen to be the middle value in the zone. If the input value is also at the middle of the zone, there is no quantization error; otherwise, there is an error. In the previous example, the normalized amplitude of the third sample is 3.24, but the normalized quantized value is 3.50. This means that there is an error of +0.26. The value of the error for any sample is less than $\Delta/2$. In other words, we have $-\Delta/2 \le \text{error} \le \Delta/2$.

The quantization error changes the signal-to-noise ratio of the signal, which in turn reduces the upper limit capacity according to Shannon.

It can be proven that the contribution of the quantization error to the SNR_{dB} of the signal depends on the number of quantization levels L, or the bits per sample n_b, as shown in the following formula:

$$SNR_{dB} = 6.02 n_b + 1.76 \text{ dB}$$

Example 4.12

What is the SNR_{dB} in the example of Figure 4.26?

Solution

We can use the formula to find the quantization. We have eight levels and 3 bits per sample, so $SNR_{dB} = 6.02(3) + 1.76 = 19.82$ dB. Increasing the number of levels increases the SNR.

Example 4.13

A telephone subscriber line must have an SNR_{dB} above 40. What is the minimum number of bits per sample?

Solution

We can calculate the number of bits as

$$\text{SNR}_{dB} = 6.02nb + 1.76 :::: 40 \longrightarrow n :::: 6.35$$

Telephone companies usually assign 7 or 8 bits per sample.

Uniform Versus Nonuniform Quantization For many applications, the distribution of the instantaneous amplitudes in the analog signal is not uniform. Changes in amplitude often occur more frequently in the lower amplitudes than in the higher ones. For these types of applications it is better to use nonuniform zones. In other words, the height of Δ is not fixed; it is greater near the lower amplitudes and less near the higher amplitudes. Nonuniform quantization can also be achieved by using a process called companding and expanding. The signal is companded at the sender before conversion; it is expanded at the receiver after conversion. Companding means reducing the instantaneous voltage amplitude for large values; expanding is the opposite process. Companding gives greater weight to strong signals and less weight to weak ones. It has been proved that nonuniform quantization effectively reduces the SNR_{dB} of quantization.

Encoding

The last step in PCM is encoding. After each sample is quantized and the number of bits per sample is decided, each sample can be changed to an llb-bit code word. In Figure 4.26 the encoded words are shown in the last row. A quantization code of 2 is encoded as 010; 5 is encoded as 101; and so on. Note that the number of bits for each sample is determined from the number of quantization levels. If the number of quantization levels is L, the number of bits is *llb* = log2 L. In our example L is 8 and *llb* is therefore 3. The bit rate can be found from the formula

$$\text{Bit rate} :::: \text{sampling rate x number of bits per sample} :::: f_s \text{ x } nb$$

Example 4.14

We want to digitize the human voice. What is the bit rate, assuming 8 bits per sample?

Solution

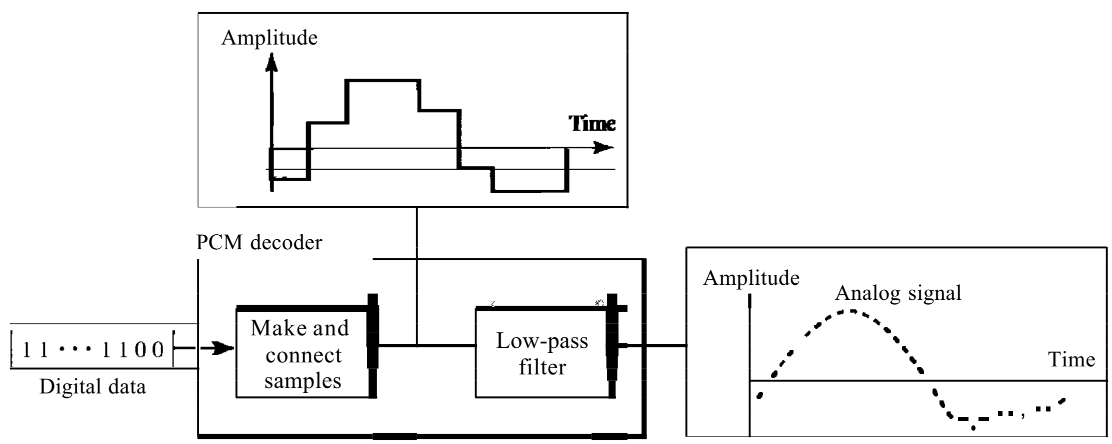
The human voice normally contains frequencies from 0 to 4000 Hz. So the sampling rate and bit rate are calculated as follows:

Sampling rate :::: 4000 x 2 :::: 8000 samples/s
Bit rate = 8000 x 8 :::: 64,000 bps = 64 kbps

Original Signal Recovery

The recovery of the original signal requires the PCM decoder. The decoder first uses circuitry to convert the code words into a pulse that holds the amplitude until the next pulse. After the staircase signal is completed, it is passed through a low-pass filter to smooth the staircase signal into an analog signal. The filter has the same cutoff frequency as the original signal at the sender. If the signal has been sampled at (or greater than) the Nyquist sampling rate and if there are enough quantization levels, the original signal will be recreated. Note that the maximum and minimum values of the original signal can be achieved by using amplification. Figure 4.27 shows the simplified process.

Figure 4.27 *Components of a PCM decoder*



PCM Bandwidth

Suppose we are given the bandwidth of a low-pass analog signal. If we then digitize the signal, what is the new minimum bandwidth of the channel that can pass this digitized signal? We have said that the minimum bandwidth of a line-encoded signal is $B_{min} = c \times N \times (1/r)$. We substitute the value of N in this formula:

$$B_{min} = c \times N \times \frac{1}{r} = c \times n_b \times f_s \times \frac{1}{r} = c \times n_b \times 2 \times B_{analog} \times \frac{1}{r}$$

When $1/r = 1$ (for a NRZ or bipolar signal) and $c = (1/2)$ (the average situation), the minimum bandwidth is

$$B_{min} = n_b \times B_{analog}$$

This means the minimum bandwidth of the digital signal is n_b times greater than the bandwidth of the analog signal. This is the price we pay for digitization.

Example 4.15

We have a low-pass analog signal of 4 kHz. If we send the analog signal, we need a channel with a minimum bandwidth of 4 kHz. If we digitize the signal and send 8 bits per sample, we need a channel with a minimum bandwidth of 8 x 4 kHz = 32 kHz.

Maximum Data Rate ofa Channel

In Chapter 3, we discussed the Nyquist theorem which gives the data rate of a channel as $N_{max} = 2 \times B \times \log_2 L$. We can deduce this rate from the Nyquist sampling theorem by using the following arguments.

1. We assume that the available channel is low-pass with bandwidth *B.*
2. We assume that the digital signal we want to send has L levels, where each level is a signal element. This means $r = 1/10g_2 L$.
3. We first pass the digital signal through a low-pass filter to cut off the frequencies above *B* Hz.
4. We treat the resulting signal as an analog signal and sample it at 2 x *B* samples per second and quantize it using *L* levels. Additional quantization levels are useless because the signal originally had L levels.

S. The resulting bit rate is $N = fs \times nb = 2 \times B \times \log2 L$. This is the maximum bandwidth; if the case factor *c* increases, the data rate is reduced.

$$N_{max} = 2 \times B \times \text{logzL} \quad \text{bps}$$

Minimum Required Bandwidth

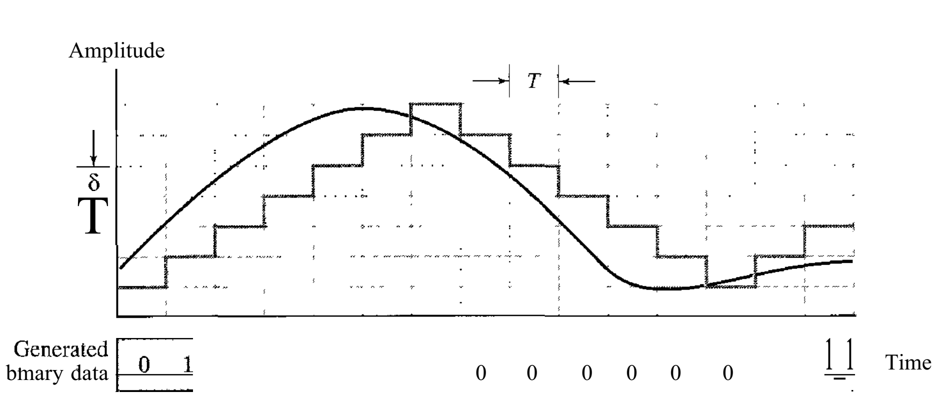
The previous argument can give us the minimum bandwidth if the data rate and the number of signal levels are fixed. We can say

$$B_{mm} = \frac{N}{2 \times \log_z L} \quad \text{Hz}$$

Delta Modulation (DM)

PCM is a very complex technique. Other techniques have been developed to reduce the complexity of PCM. The simplest is *delta modulation.* PCM finds the value of the signal amplitude for each sample; DM finds the change from the previous sample. Figure 4.28 shows the process. Note that there are no code words here; bits are sent one after another.

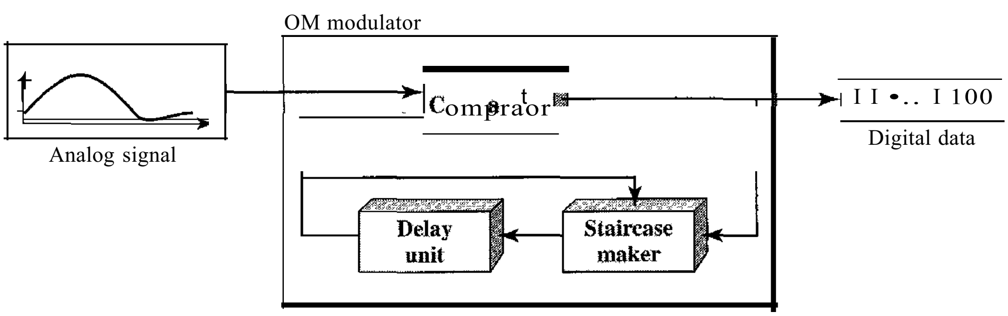
Figure 4.28 *The process ofdelta modulation*



Modulator

The modulator is used at the sender site to create a stream of bits from an analog signal. The process records the small positive or negative changes, called delta δ. If the delta is positive, the process records a I; if it is negative, the process records a 0. However, the process needs a base against which the analog signal is compared. The modulator builds a second signal that resembles a staircase. Finding the change is then reduced to comparing the input signal with the gradually made staircase signal. Figure 4.29 shows a diagram of the process.

Figure 4.29 *Delta modulation components*



The modulator, at each sampling interval, compares the value of the analog signal with the last value of the staircase signal. If the amplitude of the analog signal is larger, the next bit in the digital data is 1; otherwise, it is 0. The output of the comparator, however, also makes the staircase itself. If the next bit is I, the staircase maker moves the last point of the staircase signal δ up; it the next bit is 0, it moves it δ down. Note that we need a delay unit to hold the staircase function for a period between two comparisons.

Demodulator

The demodulator takes the digital data and, using the staircase maker and the delay unit, creates the analog signal. The created analog signal, however, needs to pass through a low-pass filter for smoothing. Figure 4.30 shows the schematic diagram.

Figure 4.30 *Delta demodulation components*