PROBABILITY

PROBABILITY(Basic Terminology)

Random Experiment: If in each trial of an experiment conducted under identical conditions, the outcome is not unique, but may be any one of the possible outcomes, then such an experiment is called random experiment e.g. tossing a coin, throwing a dice etc.

Outcome: The result of a random experiment is called an outcome

Sample Space: The set of all possible outcomes is called as sample space. It is denoted by S or Ω.

Trial: Any particular performance of a random experiment is called a trial

Event: The set of desired outcomes is called as event. It is denoted by A, B, C etc.

Exhaustive Events: The total number of possible outcomes of a random experiment is known as exhaustive events e.g. in tossing of a coin, there are two exhaustive events head and tail.

PROBABILITY(Basic Terminology)

Favorable Events: The number of cases favorable to an event in a trial is the number of outcomes which entail the happening of the event e.g. in drawing a card from a pack of cards the number of cases favorable drawing of an ace is 4

Mutually Exclusive Events: Events are said to be mutually exclusive if the happening of any one of them precludes the happening of all the others e.g. in tossing a coin the events head and tail are mutually exclusive.

Equally likely Events: Outcomes of trial are said to be equally likely if taking into consideration all the relevant evidences, there is no reason to expect one in preference to the others e.g. in throwing an unbiased die, all the six faces are equally likely to come.

PROBABILITY(Basic Terminology)

Independent Events: Several events are said to be independent if the happening of an event is not affected by the supplementary knowledge concerning the occurrence of any number of the remaining events e.g. when a die is thrown twice, the result of the throw does not affect the result of the second throw.

Probability Of an event: If a random experiment results in 'n' exhaustive, mutually exclusive and equally likely outcomes, out of which m are favorable to the occurrence of an event E, then the probability 'p' of occurrence of E, usually dented by P(E) is given by:

P = P(E) = Number of favorable cases/ Total number of exhaustive cases = m/n

PROBABILITY

- **Product Rule:** Suppose a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the 1^{st} task and n_2 ways to do the 2^{nd} task after the first task has been done, then there are $n_1 * n_2$ ways to do the procedure.

- **Sum Rule**: If a first task can be done in n_1 ways and a second task in n_2 ways and if these tasks cannot be done at the same time, then there are $n_1 + n_2$ ways to do either task.
- P (Occurrence) + P(Non Occurrence) = 1 i.e. $P(A) + P(\tilde{A}) = 1$

  $0 \leq P(A) \leq 1$

PROBABILITY PROBLEMS

- **Example:** What is the chance that a leap year selected at random will contain 53 Sundays?
- **Example**: Two unbiased dice are thrown. Find the probability that:
- i) both the dice show the same number
- ii)the first die shows 6
- iii) the total of the numbers on the dice is 8
- iv) the total of the numbers on the dice is greater than 8
- v) the total of the numbers on the dice is 13
- vi) the total of the numbers on the dice is any number from 2 to 12, both inclusive.
- **Example**: Among the digits 1,2,3,4,5 at first one is chosen and then a second selection is made among the remaining four digits. Assuming that all twenty possible outcomes have equal probabilities, find the probability that an odd digit will be selected i) the first time ii) the second time and iii) both times.

PROBABILITY PROBLEMS

- **Example:** From 25 tickets, marked with first 25 numerals, one is drawn at random. Find the chance that i) it is multiple of 5 or 7 and ii) it is a multiple of 3 or 7
- **Example**: Four cards are drawn at random from a pack of 52 cards. Find the probability that i) they are a king, a queen, a jack and an ace ii) Two are kings and two are queens iii) Two are black and two are red iv) There are two cards of hearts and two cards of diamonds.
- **Example**: In shuffling a pack of cards, four are accidently dropped, find the chance that the missing cards should be one from each suit.
- **Example**: What is the probability of getting 9 cards of the same suit in one hand at a game of bridge?
- **Example**: A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, 2 officers of the sales department and 1 chartered accountant. Find the probability of forming the committee in the following manner : i) There must be one from each category ii)It should have at least one from the purchase department iii) the chartered accountant must be in the committee.

PROBABILITY PROBLEMS

- **Example:** An urn contains 6 white, 4 red and 9 black balls. If 3 balls are drawn at random, find the probability that i) two of the balls are white ii) one is of each color iii) none is red iv) at least one is white.
- **Example**: In a random arrangement of the letters of the word 'COMMERCE', find the probability that all the vowels come together.
- **Example**: What is the probability that 4 S's come consecutively in 'MISSISSIPPI'?
- **Example**: 25 books are placed at random in a shelf. Find the probability that a particular pair of books shall be i) always together ii) never together.
- **Example**: n persons are seated on n chairs at a round table. Find the probability that two specific persons are sitting next to each other.

PROBABILITY PROBLEMS

- **Example:** A five figure number is formed by the digits 0,1,2,3,4 (without repetition). Find the probability that the number formed is divisible by 4.
- **Example:** 12 balls are distributed at random among three boxes. What is the probability that the first box will contain 3 balls?
- **Example:** I f n biscuits be distributed among N persons, find the chance that a particular person receives r (<n) biscuits.
- **Example:** What is the probability that at least two out of n people have the same birthday?

PROBABILITY PROBLEMS

- **Example:** A box contains 2 white socks and 2 blue socks. Two are drawn at random. Find the probability that they match.
- **Example:** A box with 15 integrated chips contains 5 defective. If random samples of 3 chips are drawn, what is the probability that all three are defective?
- **Example:** 5 horses are in race. Audrey picks 2 of the horses at random and bets on them. Find the probability that Audrey picked the winner.
- **Example**: A room Has 4 lamp sockets for which 4 bulbs are chosen from a group of 8 working and 5 non-working bulbs. What is the probability that the room is lit.
- **Example:** A box has 75 good IC chips and 25 defective chips. If 12 IC are selected at random, find the probability that at least 1 chip is defective?
- **Example:** Find the probability that randomly chosen 3 – letter sequence will not have any repeated letters.

PROBABILITY PROBLEMS

- **Example:** Consider a pool of 6 I/O buffers. Assume that any buffer is just a likely to be available (or occupied) as any other. Compute the probability associated with the following events:

  A = "At least 2 but no more than 5 buffers occupied"

  B = "At least 3 but no more than 5 buffers occupied"

  C = "All buffers available or an even number of buffers occupied". Also determine the probability that at least one of the events A, B and C occur.
- **Example:** A series of n jobs arrive at a computing center with n processors. Assume that each of the n^n possible assignment vectors (processor for job 1, ….., processor for job n) is equally likely. Find the probability that exactly on processor will be idle?

PROBABILITY PROBLEMS

- **Example:** If a three digit decimal number is chosen at random, find the probability that exactly k digits are ≥5, for 0 ≤ k ≤3
- **Example 35:** Three companies A, B, C are the participants in a race. A and B are equally likely to win the race, while C is twice likely to win as A is. If one and only one participates can win the race, then what is the probability that i) A wins ii) A or B wins?

PROBABILITY (LESSON NO 2)

PROBABILITY

- **Addition Theorem of probability:** If A and B are two events of a sample space S then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
- **Mutually Exclusive Events:** If A and B are two events of a sample space S such that $A \cap B = ø$ then A and B are said to be mutually exclusive events. If A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B)$
- A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. Find P(A) given that : $P(B) = 3/2\, P(A)$ and $P(C) = \frac{1}{2} P(B)$

PROBABILITY

- Given A, B, C are mutually exclusive events, explain whether the following are permissible assignments of probabilities.

  $P(A) = 0.4$, $P(B) = 0.2$, $P(C) = 0.2$, $P(A \cup B) = 0.5$

  $P(A) = 0.4$, $P(B) = 0.45$, $P(C) = 0.3$

  $P(A) = 0.6$, $P(A \cap B) = 0.5$
- The probability that a student passes a Physics test is 2/3 and the probability that he passes both a physics test and an English test is 14/45. the probability that he passes at least one test is 4/5. What is the probability that he passes the English test?
- The probability of occurrence of an event A is 0.7, the probability of occurrence of B is 0.4 and that of at least one of A and B not occurring is 0.6. Find the probability that at least A and B occurs.

PROBABILITY

- An MCA applies for two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected at Y is 0.5. The probability of at least one of his application being rejected is 0.6. What is the probability that he will be selected in one of the firms?
- Suppose A and B are events with P (A) = 0.6, P (B) = 0.3 and P ($A \cap B$) = 0.2. Find the probability that i) A does not occur, ii) B does not occur, iii) A or B occurs, iv) neither A nor B occurs.
- The probability that a certain film gets award for its story is 0.23, it will get award for its music is 0.15 and it will get award for both is 0.07. What is the probability that film will get award for a) at least one of the two b) exactly one of the two.

PROBABILITY

- The probability that a person stopping at a petrol pump will ask for petrol is 0.8, will ask for water is 0.7 and for both is 0.65. Find the probability that a person will ask for a) either petrol or water b) only water.
- Sample survey was taken to check which newspaper people read (A, B, C). In a sample of 100 people the following results are obtained. 60 read A, 40 read B, 70 read C, 45 read A and C, 32 read A and B, 38 read B and C, 30 read A, B and C. If a person is selected at random, find the probability that a) he reads only A b) he reads at least two newspapers c) he doesn't read any paper.

PROBABILITY

- Three newspapers A, B and C are published in a certain city. It is estimated from a survey that of the adult population : 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find what percentage read at least one of the newspapers?
- A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Conditional Probability

PROBABILITY

- **Conditional Probability:** If A and B are the events of sample space S, then conditional probability of A given B is the probability of A such that B has already occurred and it is given by
  $P(A|B) = P(A \cap B) / P(B)$

- **Multiplication Theorem of Probability:** If A and B are two events of a sample space S then
  $P(A \cap B) = P(A|B) \times P(B)$

- **Independent Events:** If A and B are two events of a sample space S such that $P(A|B) = P(A)$ then A and B are said to be independent events.
  If A and B are independent events then $P(A \cap B) = P(A) \times P(B)$
  If A and B are independent events then $P(A \mid \overline{B}) = P(A \mid B) = P(A)$

PROBABILITY

- Consider 4 computer firms A, B, C and D bidding for a contract. A survey of past bidding success of this firm on similar contract gives following probability of winning. P (A) = 0.35, P (B) = 0.15, P(C) = 0.3, P (D) = 0.2. Before the decision is made to avoid a contract, firm B withdraws its bid. Find the new probabilities of winning the bid for A, C and D.
- Let A and B be two events with P (A) = 3/8, P (B) = 5/8 and P (A U B) = $\frac{3}{4}$. Find P (A| B) and P (B| A).
- We are given a box containing 5000 IC chips, of which 1000 are manufactured by company X and rest by company Y. 10% of the chips made by company X and 5% of the chips made by company Y are defective. If a randomly chosen chip is found to be defective, find the probability that it comes from company X.

PROBABILITY

- Prove that with example that any events may be pair wise independent but need not to be mutually independent.
- Show that mutual independence does not imply pair wise independence.
- An electronic device is made up of 2 components A and B and is such that it works as long as 1 component works. Probability of failure of A is 0.02 and that of B is 0.1. If they work independently, find the probability that device works.
- In a certain college 25% of the students failed in Mathematics, 15% failed in Chemistry and 10% failed in both Mathematics and Chemistry. A student is selected at random

  If the student is failed in chemistry, what is the probability that he or she is failed in Mathematics?

  What is the probability that the student is failed neither in Mathematics nor in Chemistry?

PROBABILITY

- If four squares are chosen at random on a chessboard, find the chance that they should be in a diagonal line.
- A consignment of 15 record players contains 4 defectives. The record players are selected at random, one by one, and examined. Those examined are not put back. What is the probability that the 9th one examined is the last defective?
- If A and B are two events, prove that $P(A|\sim B) = P(A) - P(A \cap B) / 1 - P(B)$

Probability (Lesson 5)

BAYE'S THEOREM

Probability

- **<u>Baye's Theorem: </u>Statement: -** An event A can occur only if one of the mutually exclusive, disjoint and exhaustive set B_1, B_2, ….. , B_n occurs. Suppose that the unconditional probabilities P (B_1), P (B_2), ….. , P (B_n) and the conditional probabilities P (A|B_1), P (A|B_2), ….. , P (A|B_n) are known. Then the conditional probability P (B_i|A) of a specified event B_i when A is stated to have actually occurred is given by

$$P(B_i|A) = [P(B_i) * P(A|B_i)]/[\sum P(B_i) * P(A|B_i)]$$

This is known as **Baye's Theorem**.

Probability

- In a bolt factory machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4%, 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machines A, B and C?

Probability

- Three machines A, B and C produce respectively 40%, 10% and 50% of the items in a factory. The % of defective items produced by the machine is respectively 2%, 3% and 4%. An item from the factory is selected at random.
- Find the probability that the item is defective.
- If the item is defective, find the probability that the item was produced by machine C.

Probability

- A given lot of IC chips contains 2% defective chips. Each chip is tested before delivery. The tester itself is not totally reliable so that:
- P ('Tester says chip is good"| " The chip is actually good") = 0.95
- P ('Tester says chip is defective"| " The chip is actually defective") = 0.94
- If a tested device is indicated to be defective what is the probability that it is actually defective?

Probability

- An export agency exports tennis balls, which are supplied by 3 manufacturers A, B and C. The balls manufactured by them contain 3%, 4% and 1% defective balls respectively of the agencies total export. 50% of balls are manufactured by A, 30% by B, 20% by C. To test the quality of the balls, one ball is selected at random and inspected. Find
- P (ball is manufactured by A and is defective)
- P (ball is manufactured by B and is defective)
- P (ball is manufactured by C and is defective)
- P (ball is defective)
- P (ball manufactured by A given that it is defective

Probability

- Suppose that a product is produced in three factories X, Y and Z. It is known that factory X produces thrice as many items as factory Y and that factories Y and Z produce the same number of items. Assume that it is known that 3% of the items produced by each of the factories X and Z are defective while 5% of those manufactured by factory Y are defective. All the items produced in the three factories are stocked, and an item of product is selected at random
- What is the probability that this item is defective?
- If an item selected at random is found to be defective, what is the probability that it was produced by factory X, Y and Z respectively?

Probability

- Of all graduate students in university 70% are women and 30% are men. Suppose that 20% and 25% of the female and male population respectively smokes cigarettes. What is the probability that a randomly selected graduate is a) a women who smokes? b) A man who smokes? c) A smoker?

Probability

- **Example:** In the year 20005 there were three candidates for the position of principal Mr. Chatterjee, Mr. Iyangar and Mr. Wagh. Their chances of getting the appointment are in the proportion 4:2:3 respectively. The probability that Mr. Chatterjee is selected would introduce computer education in the college is 0.3. The probability of Mr. Iyangar and Mr. Wagh doing the same are respectively 0.5 and 0.8. What is the probability that there was computer education in the college in 2006?

Probability

- The contents of urns I, II and III are as follows: 1 white, 2 black and 3 red balls, 2 white, 1 black and 1 red balls and 4 white, 5 black and 3 red balls. One urn is chosen at random and two balls are drawn. They happen to be white and red. What is the probability that they come from I, II or III?

Probability

- There are three boxes. Box I contains 1 white, 2 red and 3 black balls. Box II contains 2 white, 3 red and 1 black balls. Box III contains 3 white, 1 red and 2 black balls. A box is chosen at random. If the balls are drawn are first red and second white, what is the probability that they come from Box II?

Probability

- A lot of transistors contain 0.6% defective. Each transistor is subjected to a test that correctly identifies a defective but also misidentifies as defective about 2 in every 100 good transistors. Given that a randomly chosen transistor is declared defective by tester, compute the probability that is actually defective.

Probability

- A binary communication channel carries data as one of two types of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as 1 and a transmitted 1 is sometimes received as 0. For a given channel, assume a probability of 0.94 that a transmitted 0 is correctly received as a 0 and a probability of 0.91 that a transmitted 1 is received as a 1. Further assume a probability of 0.45 of transferring a 0. If a signal is sent, determine:
- Probability that 1 is received.
- Probability that 0 is received.
- Probability that 1 was transmitted, given that 1 was received.
- Probability that 0 was transmitted, given that 0 was received.
- Probability of an error.

Probability

- Consider a trinary communication channel whose channel diagram is shown below. For i = 1, 2, 3, let T_i denote the event "Digit i is transmitted" and let R_i denote the event " Digit i is received'. Assume that a 3 is transmitted three times more frequently than a 1, and a 2 is sent twice as often as 1. If a 1 has been received , what is the expression for the probability that a 1 was sent? Derive an expression for the probability of a transmission error.

THANK YOU