(U8) (1) Unit-7

Fuzzy Rule Base and Approximate Reasoning

§ Truth Values and Tables In Fuzzy logic

- Linguistic variables →

In our daily life words are used to describe variables.

For Ex-①

When we say "today is hot", or "today's temparature is high",

we use the word "high" to describe the variable "today's temparature".

i.e. the variable "today's temparature" takes the word "high" as its value.

② Height is a linguistic variable if it takes

Tall, Medium, Short as its values.

- Linguistic Hedges →

Hedges are entities to deal with adverb.

or we can say Hedges are adverbs

For Ex – If we view the Speed of a car as linguistic variable then its values might be

adverbs → "not slow"
→ "very slow"
→ "slightly fast"

Telling more about values → "more or less medium"

- Human knowledge is represented in term of fuzzy IF-THEN rules.

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A fuzzy IF-THEN rule is a conditional stmt expressed as.

IF <fuzzy proposition>, THEN <fuzzy proposition>

Therefore, in order to understand fuzzy IF-THEN rules, we first must know what are fuzzy propositions.

§ Fuzzy Propositions →

There are 2 types of fuzzy Propositions

1) atomic fuzzy proposition
2) Compound fuzzy proposition

1) An atomic fuzzy proposition is a single stmt -

x is A

x → where x is linguistic variable

A → is a linguistic value of x
ie: A is a fuzzy set defined on physical domain of x.

2) A compound fuzzy proposition is a composition of atomic fuzzy proposition using the connectives "and", "or" and "NOT"

which represents
- fuzzy union → "or"
- fuzzy intersection → "and"
- fuzzy complement → "NOT"

Suppose S, M, F } denotes the fuzzy sets
S → slow
M → Medium
F → Fast

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x is S
x is M } atomic fuzzy propositions
x is F

x is S or x is not M

x is not S and x is not F

(x is S and x is not F) or x is M

A fuzzy proposition is a stmt which aquires a fuzzy truth value.

∴ Let $\tilde{P}$ is a fuzzy proposition

$T(\tilde{P})$ is the truth value attached to $\tilde{P}$.

[it varies b/w 0 & 1]

Fuzzy propositions are associated with fuzzy sets. The fuzzy membership value associated with the fuzzy set $\tilde{A}$ for $\tilde{P}$ is treated as the fuzzy truth value $T(\tilde{P})$.

i.e. $T(\tilde{P}) = \mu_{\tilde{A}}(x)$ where $0 \le \mu_{\tilde{A}}(x) \le 1$

Ex →

$\tilde{P}$: Raj is honest. set of Honest $H = \{0.2/\text{Yogesh}, 1/\text{Rajesh}, 0.8/\text{Raj}\}$ (not honest, absolutely honest, partly honest)

$T(\tilde{P}) = 0.8$ if P is partly honest.

$T(\tilde{P}) = 1$ if $\tilde{P}$ is absolutely honest.

Fuzzy connectives →

similar to crisp logic, fuzzy logic supports the following connectives →

1) conjunction (∧) : X AND Y

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ii) Disjunction (∨) : X OR Y

iii) Implication or conditional (⇒) : IF X THEN Y.

iv) Bidirectional or equivalence (⇔) : X IF AND ONLY IF Y

Ex → P : A is efficient , $T(P) = 0.8$

Q : B is efficient , $T(Q) = 0.65$

i) $P \wedge Q$: A is efficient and so is B.

$$T(P \wedge Q) = \min(T(P), T(Q))$$
$$= \min(0.8, 0.65)$$
$$= 0.65$$

ii) $P \vee Q$: Either A or B is efficient.

$$T(P \vee Q) = \max(T(P), T(Q))$$
$$= \max(0.8, 0.65)$$
$$= 0.8$$

iii) $P \Rightarrow Q$: If A is efficient then B is efficient

$$T(P \Rightarrow Q) = \max(1 - T(P), T(Q)) \quad (\because P \to Q \equiv \neg P \vee Q)$$
$$= \max(1 - 0.8, 0.65)$$
$$= \max(0.2, 0.65)$$
$$= 0.65$$

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iv) Bidirection or Equivalence ($\Leftrightarrow$)

$P \Leftrightarrow Q$: A is efficient If and only if B is efficient

$T(P \Leftrightarrow Q) = \min\left[\max(1-T(P), Q), \max(1-T(Q), P)\right]$ $\left[\because P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P) = (\neg P \vee Q) \wedge (\neg Q \vee P)\right]$

$= \min\left[\max(1-0.8, 0.65), \max(1-0.65, 0.8)\right]$

$= \min\left[\max(0.2, 0.65), \max(0.35, 0.8)\right]$

$= \min(0.65, 0.8)$

$= 0.65$

§ Formation of Rules →

The general way of representing human knowledge is by forming natural language expressions.

It is given by

IF antecedant THEN consequent.
↳ It is known as IF-THEN rule-base.

These are 3 general forms

i) Assignment stmt –

Ex – y = small
color = red
outside temperature = normal

These stmts utilize "=" for assignment.

ii) Conditional stmt –

Ex –

If temperature is high THEN climate is hot.

If y is very cool THEN stop

The conditional stmts use the "IF-THEN" form

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3. Unconditional statements
They can be of form

Goto sum
stop
Divide by a
Turn the pressure low

§ Decomposition of Rules (Compound Rules)

A compound rule is a collection of many simple rules combined together.

Any compound rule structure may be decomposed and reduced to a number of simple canonical rule forms.

Following are the methods used for decomposition of compound linguistic rules into simple canonical rules-

1. Multiple conjunctive antecedents →

If x is $\tilde{A}_1$, $\tilde{A}_2$ --- $\tilde{A}_n$ THEN y is $\tilde{B}_m$
If x is $\tilde{A}_1$ AND x is $\tilde{A}_2$ --- x is $\tilde{A}_n$ THEN y is $\tilde{B}_m$

Let a new fuzzy subset $\tilde{A}_p$ defined as

$$\tilde{A}_p = \tilde{A}_1 \cap \tilde{A}_2 \cap \tilde{A}_3 \cap \tilde{A}_n$$

so, in the form of membership func^n can be represented as

$$\mu_{\tilde{A}_p}(x) = \min\left[\mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x), \mu_{\tilde{A}_n}(x)\right]$$

And the compound rule may be rewritten as

x is ... y is ...
IF $\tilde{A}_p$ THEN $\tilde{B}_m$

2) Multiple disjunctive antecedents —

If x is $\tilde{A}_1$ OR x is $\tilde{A}_2$ ---- OR x is $\tilde{A}_n$ THEN y is $\tilde{B}_m$

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Let a new fuzzy set $\underset{\sim}{A}_p$ is defined as

$$\underset{\sim}{A}_p = \underset{\sim}{A}_1 \cup \underset{\sim}{A}_2 \cup \underset{\sim}{A}_3 \cdots \cup \underset{\sim}{A}_n$$

so in the form of membership funct can be represented as

$$\mu_{\underset{\sim}{A}_p}(x) = \max\left[\mu_{\underset{\sim}{A}_1}(x), \mu_{\underset{\sim}{A}_2}(x), \cdots \mu_{\underset{\sim}{A}_n}(x)\right]$$

An the compound rule may be rewritten as

IF x is $\underset{\sim}{A}_p$ THEN y is $\underset{\sim}{B}_m$

3) Conditional stmts (with ELSE and UNLESS):

stmt of type

IF $\underset{\sim}{A}_1$ THEN ($\underset{\sim}{B}_1$ ELSE $\underset{\sim}{B}_2$)

can be decomposed into 2 simple canonical rule forms, connected by "OR"

IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_1$
OR
IF NOT $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_2$

IF $\underset{\sim}{A}_1$ (THEN $\underset{\sim}{B}_1$) UNLESS $\underset{\sim}{A}_2$

can be decomposed as

IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_1$
OR

IF $\underset{\sim}{A}_2$ THEN NOT $\underset{\sim}{B}_1$
IF $\underset{\sim}{A}_1$ THEN ($\underset{\sim}{B}_1$) ELSE IF $\underset{\sim}{A}_2$ THEN ($\underset{\sim}{B}_2$)

can be decomposed into the form

IF $\underset{\sim}{A}_1$ THEN $\underset{\sim}{B}_1$
OR
IF NOT $\underset{\sim}{A}_1$ AND IF $\underset{\sim}{A}_2$ THEN $\underset{\sim}{B}_2$

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4) Nested-IF-THEN rules:

The rule

"IF $\underset{\sim}{A}_1$ THEN [IF $\underset{\sim}{A}_2$ THEN ($\underset{\sim}{B}_1$)]"

can be of the form

IF $\underset{\sim}{A}_1$ AND $\underset{\sim}{A}_2$ THEN $\underset{\sim}{B}_1$

§ Aggregation of Fuzzy Rules →

The computation of "IF" part is aggregation & The computation of "THEN" part is composition.

If I have a rule as →

If x is $\underset{\sim}{A}$ AND y is $\underset{\sim}{B}$

I have to evaluate this rule in order to fix this in the form of forward chaining.

I have to first evaluate this antecedent

If the temperature is High Here x is temperature and $\underset{\sim}{A}$ is high set of

now suppose the temperature is 70° and in the fuzzy set high the membership value of 70° is $\mu_{high}(70) = 0.7$

So

If (x is $\underset{\sim}{A}$) AND y is $\underset{\sim}{B}$ — This will evaluate not to 1 or 0 but by 0.7

$\underset{\sim}{A} = \{0.2/20°, 0.4/30°, 0.6/50°, 0.7/70°, 1/90°\}$

70° ..... $\mu_{high}(70) = 0.7$

0.7

If the temperature is high & pressure is low

Let y is pressure say 50 PSI in the set low pressure $\underset{\sim}{B}$ some how it evaluate to 0.8 then

what will be the evaluation or truth of this entire antecedent

we actually take the minimum of these 2

$\min(0.7, 0.8) = 0.7$

So, the evaluation of conjuncted antecedent is 0.7

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§ Max - Min rule of composition →

Given N observations E_N over X and hypothesis H_N over Y we have N rules

If E_1 then H_1
If E_2 then H_2
if
⋮ ⋮ ⋮ ⋮
If E_N then H_N

$$\mu_H = \max\{\min(\mu_{E1}), \min(\mu_{E2}) \text{ ---- } \min(\mu_{EN})\}$$

To evaluate the strength of the Hypothesis, so

here → if E_1 then H_1 — It is x is A and y is B

there we took min simlarly we will take min of E_2 --- min of E_i and then out of those we will take max.

we took min of these 2 (0.7)

0.7 This one evaluated to 0.7 — This one evaluated to 0.8

These are 2 rules.

1. x is A and y is B ⟹ P is Q
   0.7 — 0.8
2. j is X and k is Z ⟹ R is L
   0.5 — 0.9 this evaluated to 0.9

suppose evaluated to 0.5 — min of these is 0.5

Then we will take max of 0.5 & 0.7

so max (0.7, 0.5) = 0.7

so, we will select 1st rule and we will fire P is Q consequent.

That is the exactly what it mean —
max of all mins.

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There are 2 rules for aggregation.

1. Conjunctive system of rules –
In this case rules are to be jointly satisfied.
Here the aggregated output y is determind by the fuzzy intersection. [rules are connected with "and" connetive]

$$y = y_1 \cap y_2 \cap y_3 \cdots \cap y_n$$

This aggregated o/p is defined by the membership function.

$$\mu_{y(y)}(y) = \min[\mu_{y_1}(y), \mu_{y_2}(y) \cdots \mu_{y_n}(y)] \text{ for } y \in Y$$

2. Disjunctive sys of rules →

In this case, the satisfaction of at least one rule is required. [rule are connected with "OR" connective]
The aggregated o/p is defined by the determined ,

$$y = y_1 \cup y_2 \cup y_3 \cup \cdots \cup y_n$$

again it can be defined by the membership funcⁿ

$$\mu_y(y) = \max[\mu_{y_1}(y), \mu_{y_2}(y) \cdots \mu_{y_n}(y)] \text{ for } y \in Y$$

§ Fuzzy Reasoning [Approximate Reasoning]
Reasoning means obtaining new propositions from existing propositions.
There are 4 modes of fuzzy approximate reasoning.

1) Categorical reasoning
2) Qualitative reasoning
3) syllogistic reasoning
4) Dispositional reasoning

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§ Fuzzy Inference Systems →

Fuzzy Inference systems are also known as

Fuzzy Rule-Based system

Fuzzy Models

Fuzzy Associative Memories (FAM) or

Fuzzy Controller

Basically FIS is composed of 5 functional Blocks.

1. A rule base → It contains numerous fuzzy IF-THEN rules.
2. A database → It defines the membership funcⁿ of fuzzy sets.
3. Decision-making unit → It performs operation on the rules.
4. Fuzzification interface unit → It converts the crisp quantities into fuzzy quantities.
5. Defuzzification Interface Unit → It converts the fuzzy quantities into crisp quantities.

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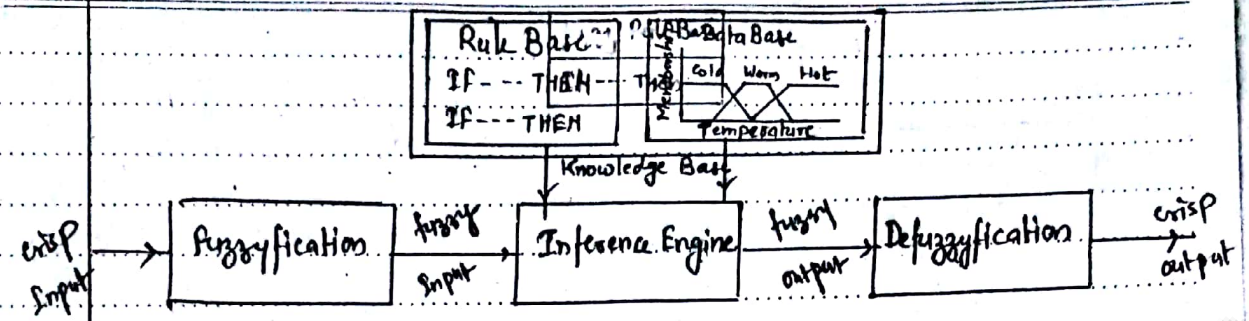

Working Methodology of FIS →

Initially - In the fuzzification unit, the crisp i/p is converted into a fuzzy input. [various fuzzification methods are employed for this]

After this process - Rule base is formed [fuzzy Rules are formed] and Knowledge Base is prepared.

↙ RuleBase + ↘ Database

Finally - defuzzification process is carried out to produce crisp output.

Methods of FIS

There are 2 important types of FIS -

1) Mamdani FIS (1975)
2) Sugeno FIS (1985)

Q Explain Mamdani type of fuzzy Inference System in detail - (8)

Q. Explain 2 types of fuzzy Inference system in detail (10)

Q. Write short note - (5)

Mamdani & Takagi - Sugeno Architecture

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§ Mamdani FIS

This system was proposed by Ebrahim Mamdani in 1975 to control a steam engine and boiler combinatn for this rules obtained from experienced people whowere working on the system.

To compute the output from Mamdani FIs following steps have to be followed

step1: Determine a set of fuzzy rules.

step2: Make the i/ps fuzzy using input membership funcns.

imp steps

step3: calculate degree of fulfillment of rules

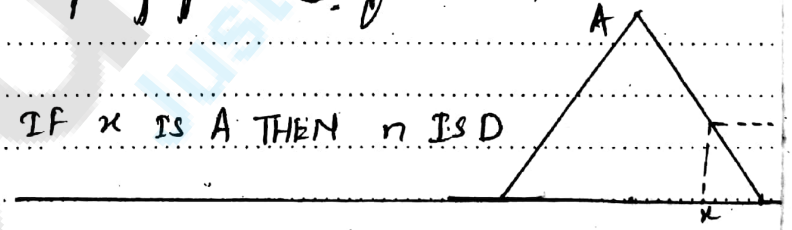
step4: Calculate fuzzy o/p of each rule

step5: Aggregate Rule outputs.

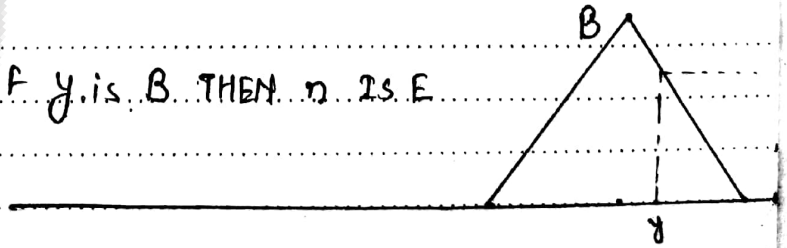
step6: Defuzzify.

Calculate degree of fulfillment of rules →

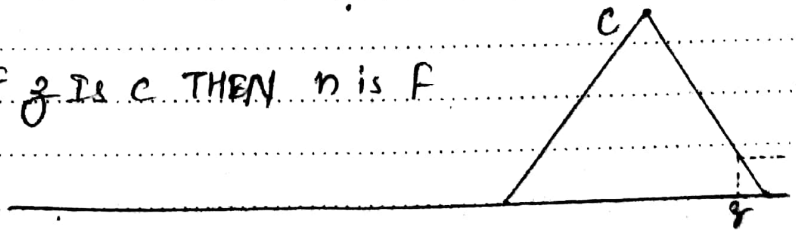
rule 1: IF x IS A THEN n IS D



rule 2: IF y is B THEN n IS E



rule 3: IF z IS C THEN n is F



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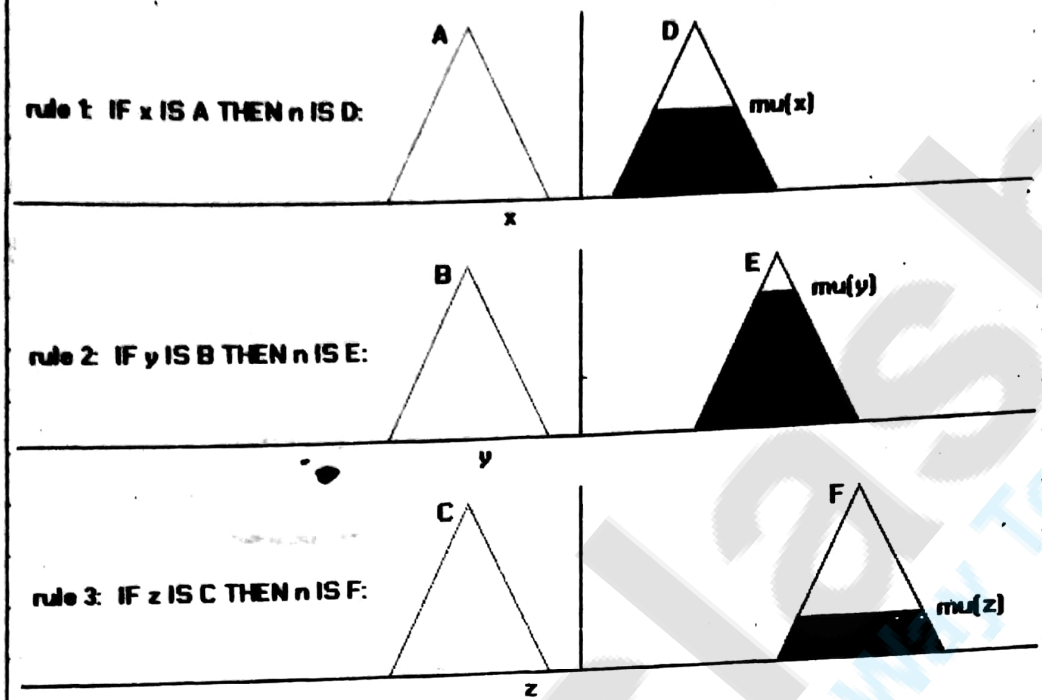
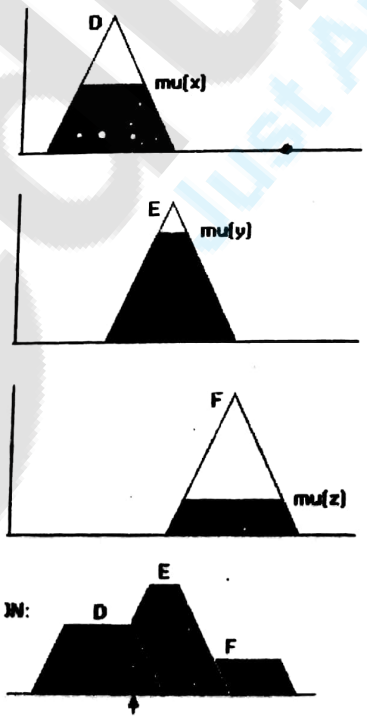
Calculate Fuzzy output of each rule →



Aggregate Rule outputs →



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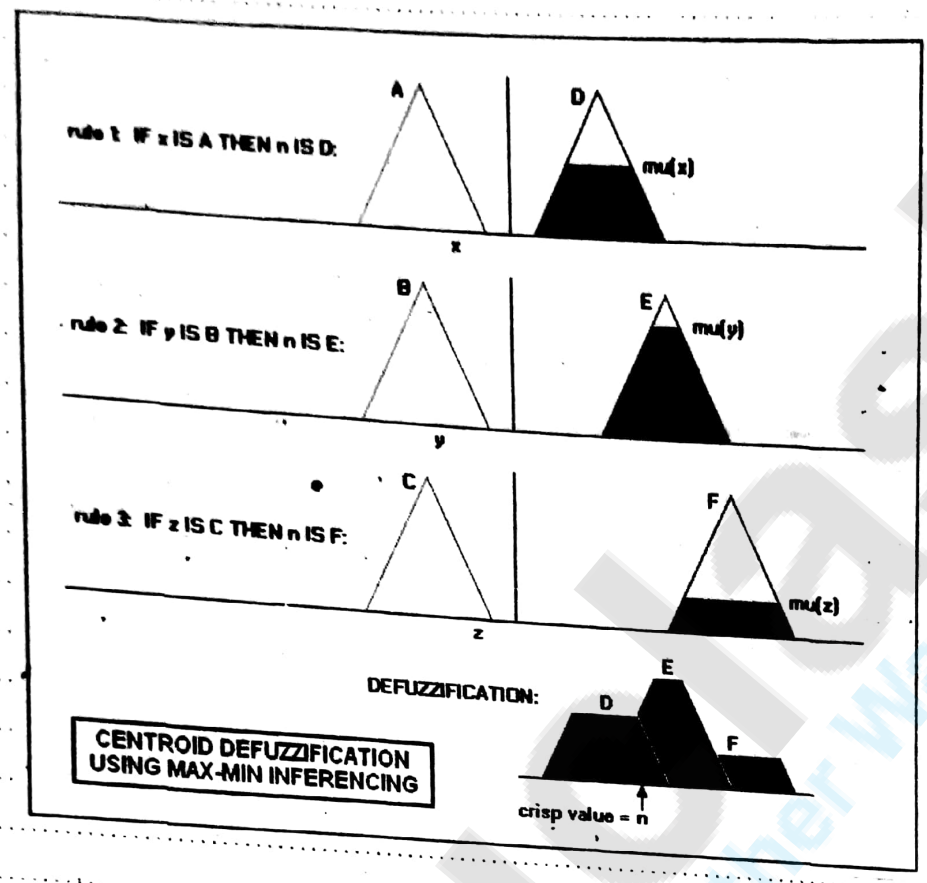
Defuzzify →



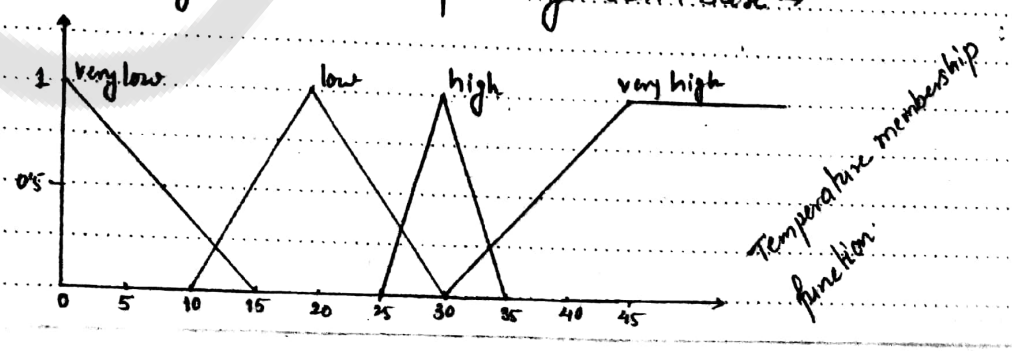
Ex- Air Conditioner System -

It consists of 2 inputs from temperature and humidity sensors providing the temperature and humidity of the room.

The Sys has 1 output that controlls the compressor speed.

The temperature and humidity are taken to be in ranges of 0°c to 45°c and 0% to 100% respectively.

The knowledge Base has following data base →



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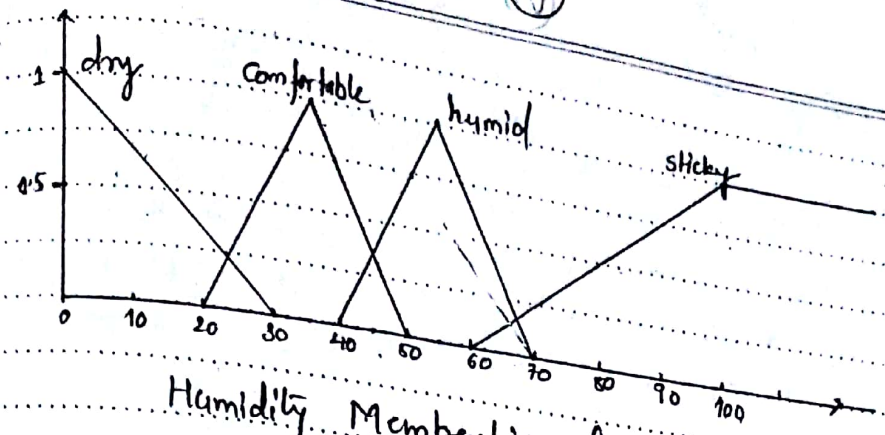
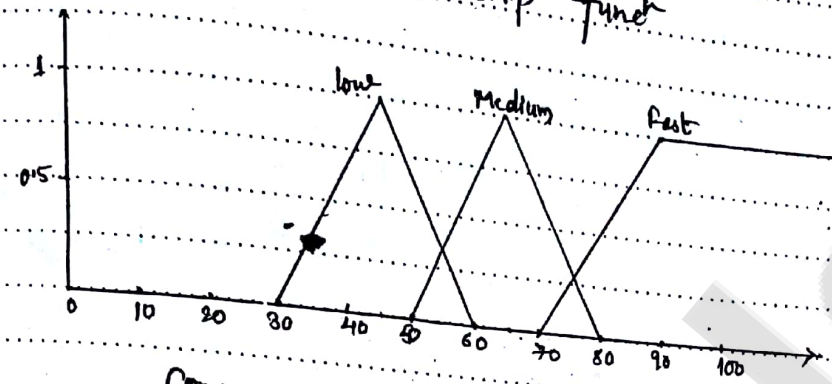


Humidity Membership funct



Compressor speed membership funct.

Rule base for Mamdani type FIs.

Rules	Temperature	Humidity	Compressor speed
1.	Very Low	Dry	Off
2.	Very Low	Comfortable	Off
3.	Very Low	Humid	Off
4.	Very Low	Sticky	Low
5.	Low	Dry	Off
6.	Low	Comfortable	Off
7.	Low	Humid	Low
8.	Low	Sticky	Medium
9.	High	Dry	Low
10.	High	Comfortable	Medium
11.	High	Humid	Fast
12.	High	Sticky	Fast
13.	Very High	Dry	Medium
14.	Very High	Comfortable	Fast
15.	Very High	Humid	Fast
16.	Very High	Sticky	Fast

Consider for 2 rules -

1. If temp is high and humidity is dry THEN Compressor speed is Low

If temp is 92° and humidity is 15 THEN Compressor speed is

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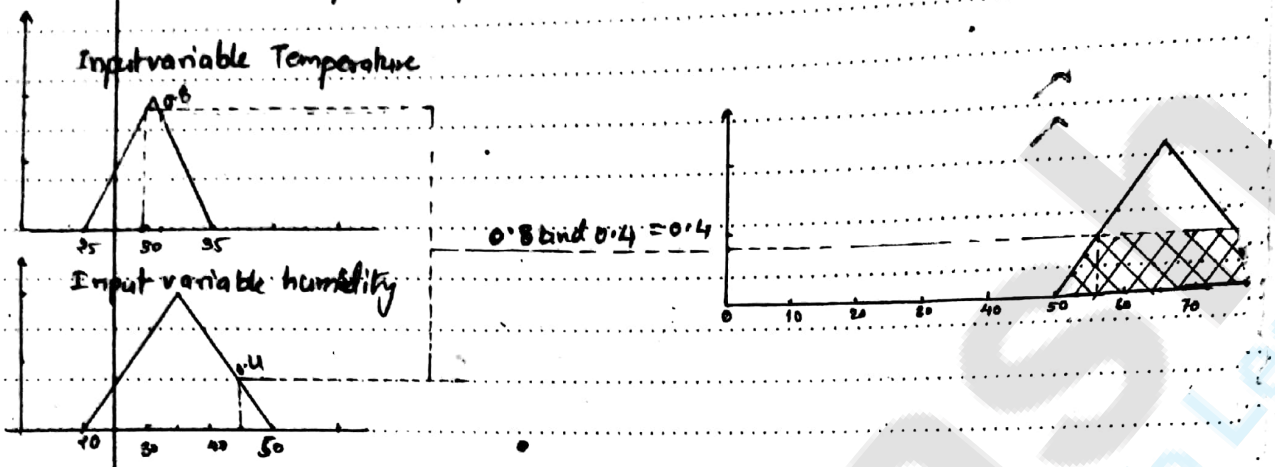
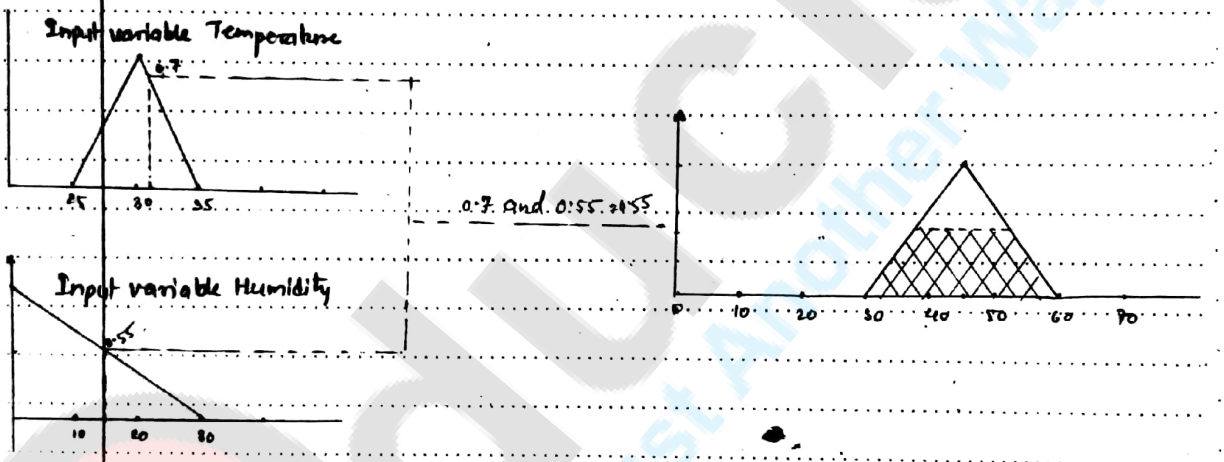
Rule 2-

If temp is high and humidity is comfortable then Compressor speed should be Medium

If temp Is 29° and humidity Is 45 THEN compressor speed shouldbe

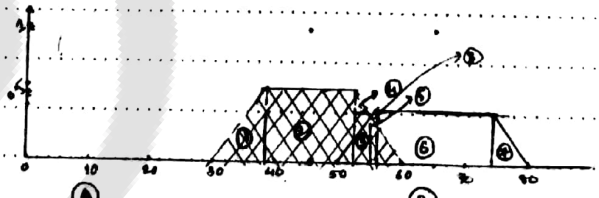


Rule 2



Rule 1

Final compressor speed Adjustment By Centroid Defuzzification →



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Region	Area	$\bar{x}(R_i)$	$(\bar{x}_i)$
1	$\frac{1}{2} \times b \times h = \frac{1}{2} \times 40 \times 0.55 =$		$= \frac{30+40+40}{3}$
2	$b \times h$ =		$= \frac{28.5+38.5+52.5+52.5}{4}$
3	$b \times h$ =		$= \frac{52.5+52.5+55.5+55.5}{4}$
4	$\frac{1}{2} b \times h$ =		$= \frac{52.5+52.5+55.5}{3}$
5	$b \times h$ =		$= \frac{55.5+55.5+52.5+52.5}{4}$

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8 Takagi - Sugeno Fuzzy Model [TS Method]

1. This method was proposed by Takagi, Sugeno and Kang in the year 1985.

2. According to TS method, the format of a Rule is given by -

IF x is $\underset{\sim}{A}$ and y is $\underset{\sim}{B}$ THEN $z = f(x,y)$

where $\underset{\sim}{A}$ & $\underset{\sim}{B}$ are fuzzy sets in antecedents

$z = f(x,y)$ is a crisp func^n in consequent

TS model replaces the fuzzy consequent (THEN part) of Mamdani model with func^n (equation) of the input variables.

Here, each Rule has a crisp output.

3. Types → There are 2 types Sugeno Fuzzy Models -

1. Zero-order - [If function f is constant]

Ex -

IF x is Small AND y is Large THEN $z = 5$

2. First-order - [If function f is a linear func^n]

Ex -

IF x is large AND Y is small THEN $z = -x + 3$

It is a special case of the Mamdani System.

4. TS model avoids the time consuming defuzzification method (centroid method) that is necessary in Mamdani Model.

TS Model uses Weighted Average Method to defuzzify.

$$z = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}$$

where w_i is matching degree of Rule R_i [result of antecedent]

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5. Diagramatic Representation of Sugeno Rule-

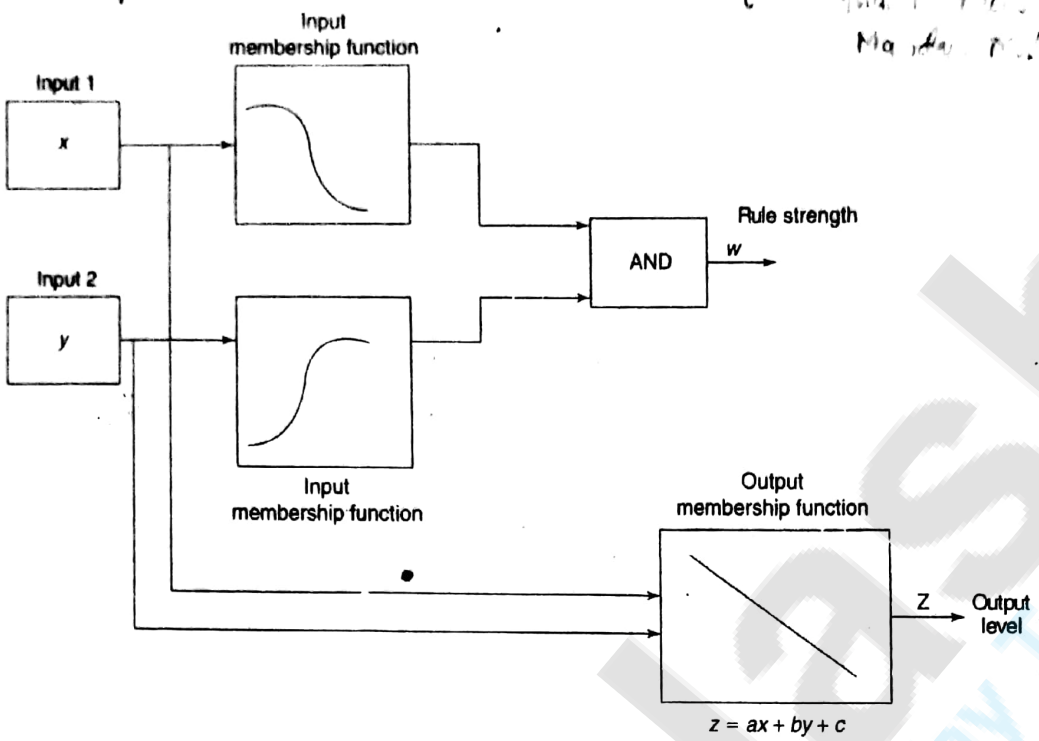
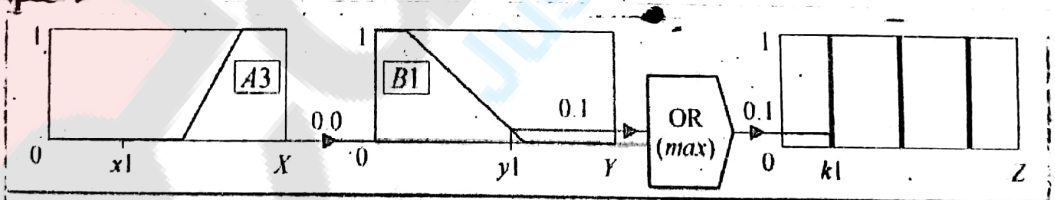


Figure 12-3 Sugeno rule.

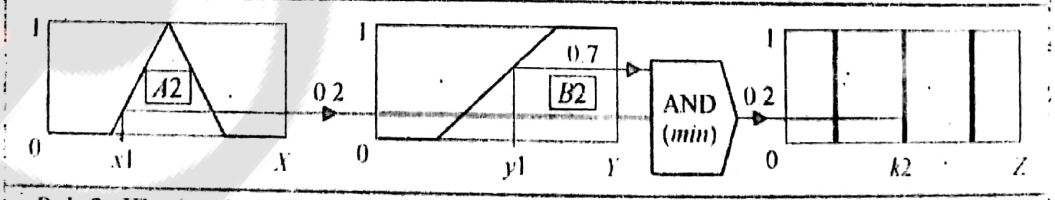
6. The main steps of the fuzzy inference process -
   - Fuzzifying the inputs &
   - Applying the fuzzy operator

are exactly same in Mamdani as well as Sugeno's Method only the difference is the Sugeno Output membership fun^{n}.

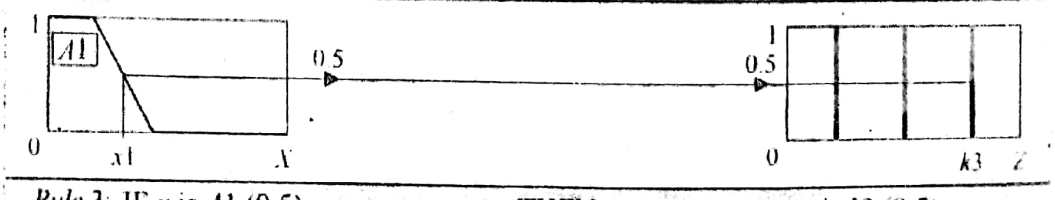
7. Example→



Rule 1: IF x is $A3$ (0.0) OR y is $B1$ (0.1) THEN z is $k1$ (0.1)



Rule 2 IF x is $A2$ (0.2) AND y is $B2$ (0.7) THEN z is $k2$ (0.2)



Rule 3: IF x is $A1$ (0.5) THEN z is $k3$ (0.5)

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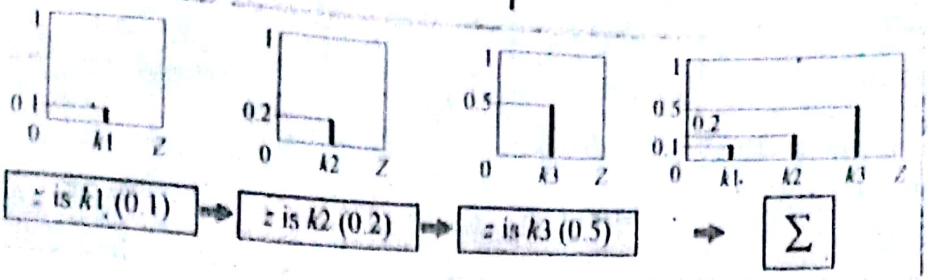
Aggregation of Rule Outputs -



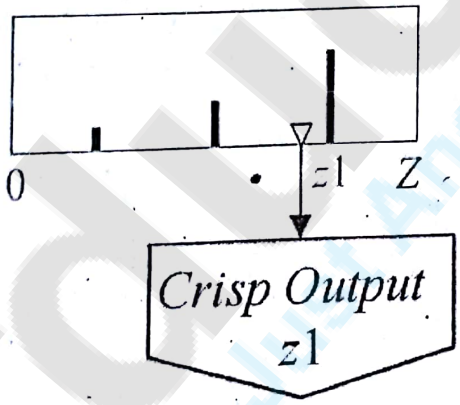
Weighted Average -

$$WA = \frac{\mu(k_1) \times k_1 + \mu(k_2) \times k_2 + \mu(k_3) \times k_3}{\mu(k_1) + \mu(k_2) + \mu(k_3)}$$

$$= \frac{0.1 \times 20 + 0.2 \times 50 + 0.5 \times 80}{0.1 + 0.2 + 0.5}$$

$$= 65$$

Defuzzification -



* Difference Between →

Mamdani	Takagi-Sugeno
1. Here, O/p membership funch is clipped at the rule strength.	1. Here, O/p membership function is either Linear or Constant
2. Here, For defuzzification centroid method is used which is	2. Here, for defuzzification weighted Average method is used.

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Mamdani	Takagi Sugeno
time consuming method.	
3. It has widespread acceptance.	3. It is computationally efficient. It is compact and work well with linear technique, optimization technique and adaptive technique
4. It is well suitable for human input.	4. It is best suited for mathematical analysis.

Note- The main problem with the Takagi-Sugeno-Kang fuzzy sys are-

a. Its THEN part is a mathematical formula and therefore may not provide a natural framework to represent human knowledge.

b. There is not much freedom left to apply different principles in fuzzy logic.

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08 27

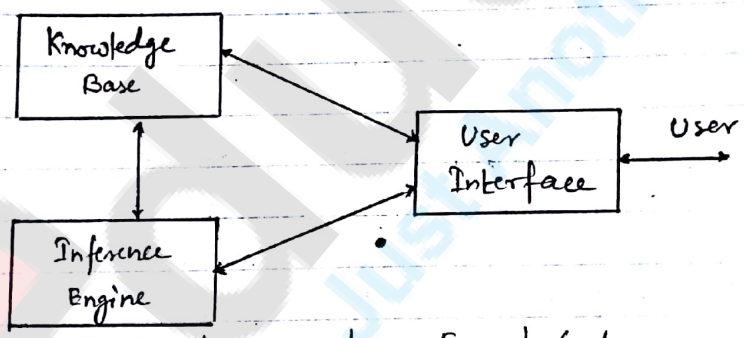
§ Overview of Fuzzy Expert System →

Expert system is the one of the facilities provided by Artificial Intelligence which can give decision or help humans to take decision for a particular problem. It has ability to simulate human thinking.
There are 2 major functions of Expert System –

1. It deals with uncertain and incomplete information
2. It interacts with user and explains how and why sys has reached at the particular decision. And gives decision also.

The Expert System has 3 major blocks –

1. Knowledge base
2. Inference Engine
3. User Interface.



Block diagram of an Expert System

1. Knowledge Base → It contains the knowledge that is specific to domain of application.

2. Inference Engine → It uses the knowledge in the knowledge base ~~for~~ and gives suitable reasoning according to the users queries.

3. User Interface → It provides smooth communication between the user and the system.

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Examples of ES →

1. MYCIN - It is an ES that is used for medical diagnosis. MYCIN rules have strength that is also called as certainty factor.
This factor lies in the unit interval [0, 1]

When traditional Expert sys uses fuzzy logic then it is called fuzzy Expert sys.

Fuzzy Expert Systems are those systems that incorporate fuzzy sets or fuzzy logic for their reasoning process and knowledge representation.

Fuzzy Expert Systems can effectively handle both uncertainty and vagueness.

Some, other examples are Z-II, MILORD, SPERIL etc.

2. SPERIL is a special fuzzy expert system that analyzes earthquake damages.

3. Z-II - is an ES shell that helps developer to represent human knowledge easily though the knowledge is uncertain and vague.
This shell is based on fuzzy logic and fuzzy numbers and it is built in minicomputer environment.

4. MILORD - is also an ES building tool (shell). It consists of two inference engines and an explanation module. And finally, an application to the diagnosis and treatment of pneumonia is presented.

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