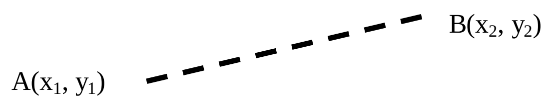
Q1(a) What are the fundamental steps in Digital Image Processing? Explain in brief. [10]
Q1(b) Explain Koch curve?. [5]
Q1(c) What is viewing pipeline ? Explain the blocks of viewing pipeline. [5]

Q2(a) Derive a Digital Differential Line drawing Algorithm to draw dash line of +ve values of slope as given blow. [7]



Hint :

Assume length of dash = **d** pixels and length of Gap between dash = **g** pixels

Assume slope of line AB ≤ 1 (for simplicity)

Develop DDA algorithm to draw line segment that will display **d** number of pixels and skip **g** number of pixels between every dash

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Q2(b) What are the properties of concatenation of transformations ? What is the sequence of transformations required to change the position of object in figure-1 to figure-2. [8]

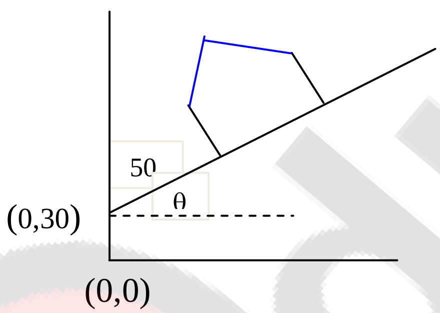


Figure-1

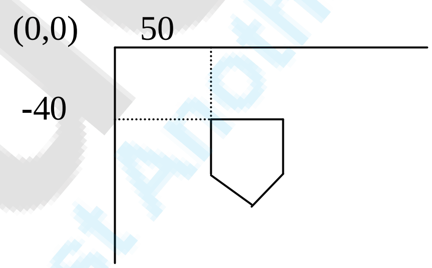


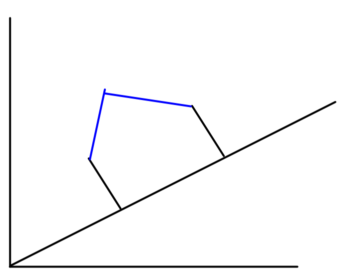
Figure-2

Theory : Properties of Concatenation.
Problem : Solution :

Step-1 : Translate by tx = 0 and ty = -30

The required Translation Transformation matrix $\mathbf{T_1}$ is given by,

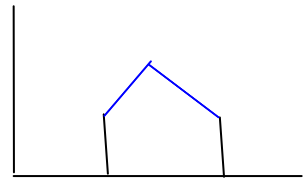
$$\mathbf{T_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -30 \\ 0 & 0 & 1 \end{bmatrix}$$



Step-2 : Rotate by θ clockwise

The required Rotation Transformation matrix R is given by,

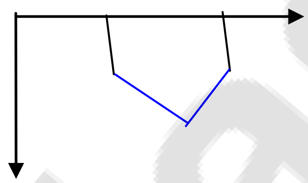
$$\mathbf{R} = \begin{bmatrix} \cos(\boldsymbol{\theta}) & \mathbf{sin(\theta)} & \mathbf{0} \\ \mathbf{-sin(\theta)} & \cos(\boldsymbol{\theta}) & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$



Step-3 : Reflect about y-axis

The required Refection Transformation matrix Ref given by,

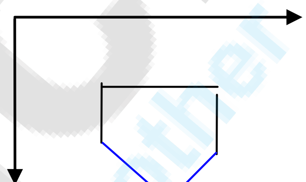
$$\mathbf{R_f} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Step-4 : Translate by tx=0 and ty= -40

The required Translation Transformation matrix $\mathbf{T_2}$ is given by,

$$\mathbf{T_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -40 \\ 0 & 0 & 1 \end{bmatrix}$$



Step-5 : Find Composite Transformation Matrix $\mathbf{T_M}$

$$\mathbf{T_M = T_2 \; R_f \; R \; T_1}$$

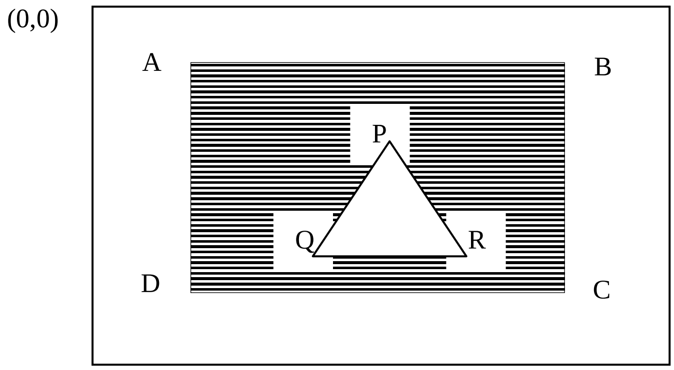
Step-6 : Find Output Object Co-ordinates

Let **A, B, C, D and E** be input object co-ordinates
AND A', B', C', D', and E' be the output object co-ordinates such that

$$[A', B', C', D', E'] = \mathbf{T_M}\,[A, B, C, D, E\,]$$

Q3(a) State Liang Barsky Line Clipping Algorithm. [7]

Q3(b) Develop an scanline polygon filling algorithm.. Explain the working of your algorithm for the following picture. The triangle PQR is cut-out from the rectangle ABCD. Fill colour for Rectangle ABCD is BLACK and for Triangle PQR is WHITE. [8]



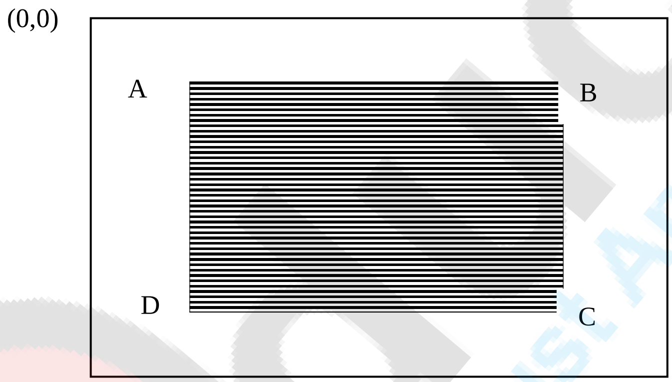
Solution :

Part-I : Scan Line Fill Algorithm

Part-II : Working of scan line fill algorithm for the given picture :

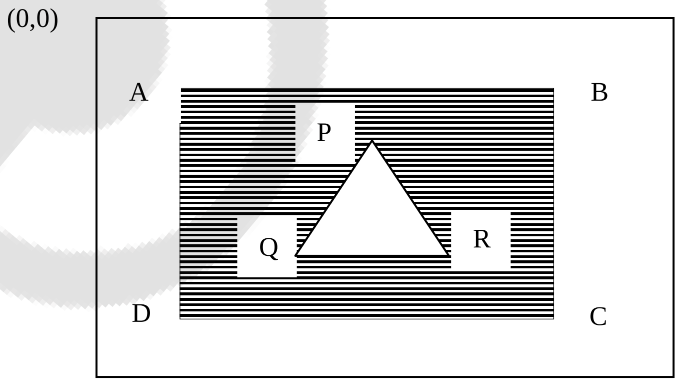
Step-1 : Set Fill Colour = BLACK
Apply scan line fill algorithm to polygon ABCD

Result of filling ABCD :



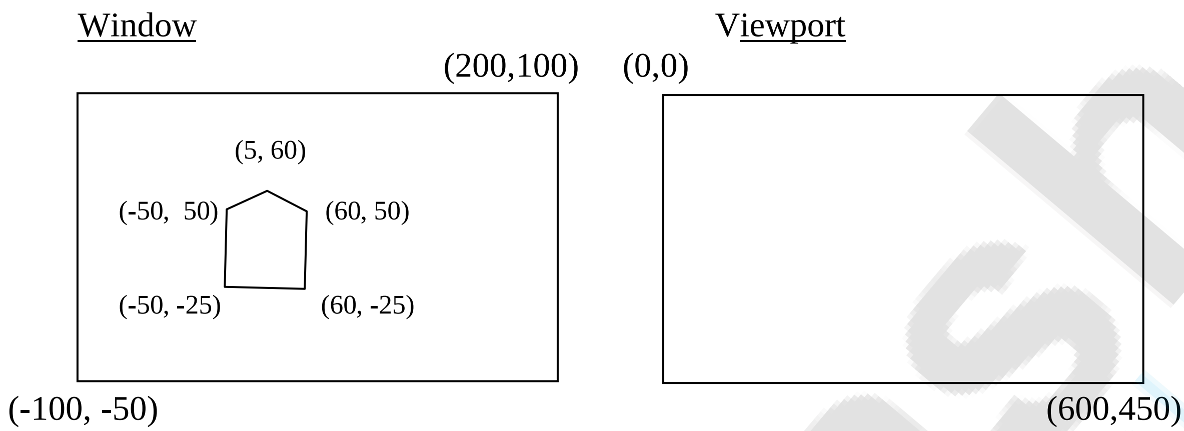
Step-2 : Set Fill Colour = WHITE
Apply scan line fill algorithm to polygon PQR

Result of filling PQR



Q4(a) Derive two dimensional rotational transformation matrix. [7]

Q4(b) Figure below depicts a picture in the "window". For the viewport shown alongside evaluate & draw the mapped picture. [8]



Solution :
In this problem, Window is defined RIGHT handed co-ordinate system and LEFT handed co-ordinate system.
This is based on Scaling Ttransformation. So, we need to calculate Scaling Transformation Parameters Sx and Sy.

Window parameters :
Xwmin = - 100, Ywmin = -50,
Xwmax = 200, Ywmax = 100 ,

Viewport parameters :
Xvmin = 0, Yvmin = 0,
Xvmax = 600, Yvmax = 450

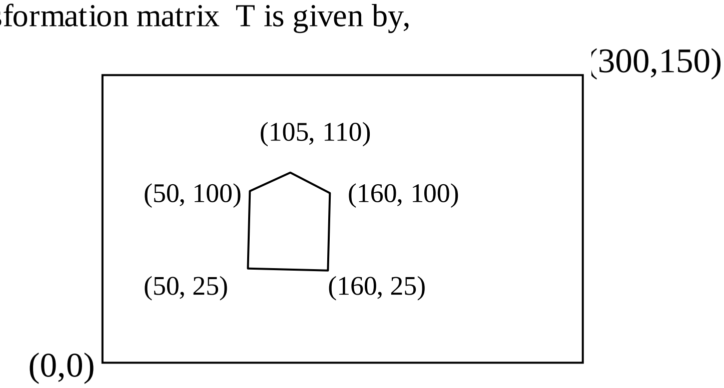
$$Sx = \frac{Xvmax - Xvmin}{Xwmax - Xwmin} = \frac{(600 - 0)}{(200 - (-100)} = 2$$

$$Sy = \frac{Yvmax - Yvmin}{Ywmax - Ywmin} = \frac{(450 - 0)}{(100 - (-50)} = 3$$

Step-1 : Translate by tx=100 and ty=50

The required Translation Transformation matrix T is given by,

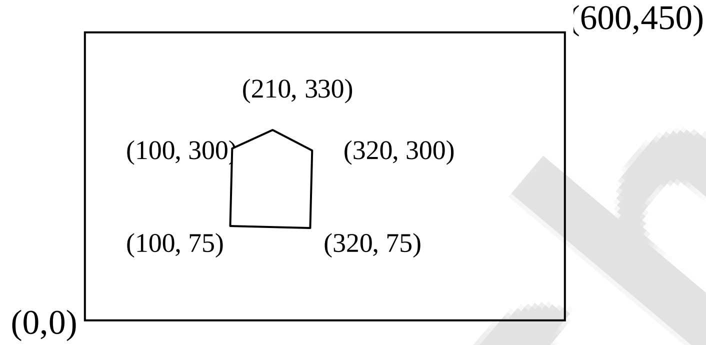
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$$



Step-2 : Scale by Sx=2 and Sy=3

The required Scaling Transformation matrix S is given by,

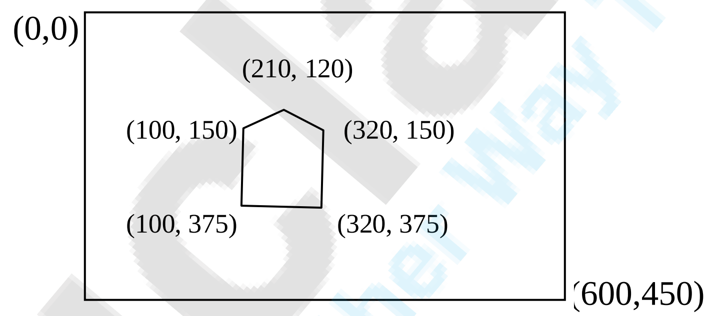
$$\mathbf{S} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Step-3 : Transform RIGHT handed Co-ordinate System to LEFT handed Co-ordinate System

The required Transformation matrix T_{RL} is given by,

$$\mathbf{T_{RL}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 450 \\ 0 & 0 & 1 \end{bmatrix}$$



Step-4 : Find Composite Transformation Matrix $\mathbf{T_M}$

$$\mathbf{T_M} = \mathbf{T_{RL}} \quad \mathbf{S} \quad \mathbf{T}$$

$$\mathbf{T_M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 450 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 450 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_M = \begin{bmatrix} 2 & 0 & 200 \\ 0 & -3 & 300 \\ 0 & 0 & 1 \end{bmatrix}$$

Step-5 : Find Output Object Co-ordinates

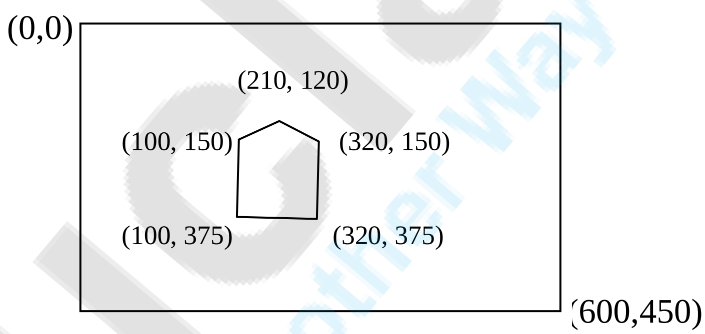
Let input object co-ordinates be **A(-50,-25) B(-50, 50), C(5, 60), D(60, 50) and E(60, -25)**

Let A', B', C', D', and E' be the output object co-ordinates such that
[A', B', C', D', E']= $\mathbf{T_M}$ [A, B, C, D, E]

$$[\mathbf{A', B', C', D', E'}] = \begin{bmatrix} 2 & 0 & 200 \\ 0 & -3 & 300 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -50 & -50 & 5 & 60 & 60 \\ -25 & 50 & 60 & 50 & -25 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$[\mathbf{A', B', C', D', E'}] = \begin{bmatrix} 100 & 100 & 210 & 320 & 320 \\ 375 & 150 & 120 & 150 & 375 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Result :



NOTE :
Since viwport is shown inverted, if it is solved by simply mapping Window to the dimension of Viewport without considering Transformation from RIGHT handed Co-ordinate System to LEFT handed Co-ordinate system, solution should be accepted.

Step-1 and Step-2 of earlier solution will remain same.

Step-3: Find Composite Transformation Matrix.

The composite transformation matrix is then given by,
$\mathbf{T_M}$ **= S T**

$$\mathbf{T_M} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_M = \begin{bmatrix} 2 & 0 & 200 \\ 0 & 3 & 150 \\ 0 & 0 & 1 \end{bmatrix}$$

Step-4 : Find Output Object Co-ordinates

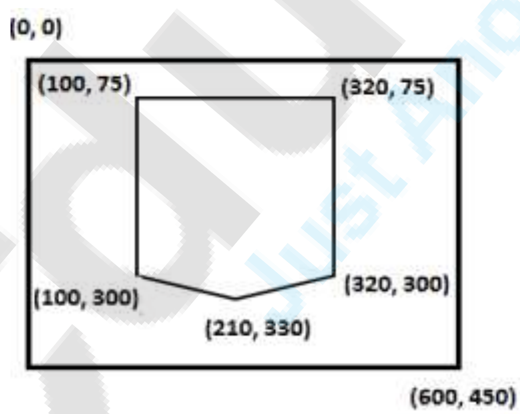
Let input object co-ordinates be **A(-50,-25) B(-50, 50), C(5, 60), D(60, 50) and E(60, -25)**

Let A', B', C', D', and E' be the output object co-ordinates such that
[A', B', C', D', E']= $\mathbf{T_M}$ [A, B, C, D, E]

$$[\mathbf{A}', \mathbf{B}', \mathbf{C}', \mathbf{D}', \mathbf{E}'] = \begin{bmatrix} 2 & 0 & 200 \\ 0 & 3 & 150 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -50 & -50 & 5 & 60 & 60 \\ -25 & 50 & 60 & 50 & -25 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$[\mathbf{A}', \mathbf{B}', \mathbf{C}', \mathbf{D}', \mathbf{E}'] = \begin{bmatrix} 100 & 100 & 210 & 320 & 320 \\ 75 & 300 & 330 & 300 & 75 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Result :



Q5(a) Explain Half-toning and Dithering technique. [7]

Q5(b) Derive perspective projection transformation of any point p(x,y,z) onto the xy plane with center of projection COP(x_p, y_p, z_p). [8]

Q6(a) Derive the sharpening second order derivative Laplacian mask in image enhancement.[7]

Derivation :

Consider a Digital sub-image as shown in fig below,

$$F = \begin{bmatrix} f(x-1,y-1) & f(x-1,y) & f(x-1,y+1) \\ f(x,y-1) & f(x,y) & f(x,y+1) \\ f(x+1,y-1) & f(x+1,y) & f(x+1,y+1) \end{bmatrix}$$

Laplacian of Digital Image is defined as $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} == G_x^2 + G_y^2$

Step-1: Find G_x^2

$$G_x = [f(x+1,y) - f(x,y)]$$

$$G_x^2 = [f(x+1,y) - f(x,y)] - [f(x,y) - f(x-1,y)]$$

$$= f(x+1,y) - 2f(x,y) + f(x-1,y)$$

Step-2: Find G_y^2

$$G_y = [f(x,y+1) - f(x,y)]$$

$$G_y^2 = [f(x,y+1) - f(x,y)] - [f(x,y) - f(x,y-1)]$$

$$= f(x,y+1) - 2f(x,y) + f(x,y-1)$$

Step-3 : Find Laplacian 4 directional mask

$$\nabla^2 f = G_x^2 + G_y^2$$

$$= f(x+1,y) - 2f(x,y) + f(x-1,y) + f(x,y+1) - 2f(x,y) + f(x,y-1)$$

$$= f(x+1,y) - 4f(x,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)$$

$$\nabla^2 f = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Scaling by (-1) we get, $\nabla^2 f = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

This is Laplacian four directional mask.

Step-4 : Find Laplacian 8 directional mask

Laplacian 8 Directional Mask = Lapacian 4 Directional Mask + Laplacian Diagonal Direction Mask

$$\nabla^2 f = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$ This is Laplacian 8 Directional Mask.

Q6(b) For the following four bit image perform the following operations :- [8]

(1) Threshold T = 8
(2) Intensity Level Slicing with background a = 6 and b = 12
(3) Median filtering only at center location. Remaining values no change.
(4) Negation

$$F = \begin{bmatrix} 2 & 13 & 4 \\ 15 & 6 & 12 \\ 0 & 9 & 3 \end{bmatrix}$$

Solution :

Q6(b)(1) Thresholding Transformation

- Thresholding Transformation is defined as,

$$S = \begin{bmatrix} L-1 & if \quad r \geq Threshold = T \\ 0 & Otherwise. \end{bmatrix}$$

For 4 bit image L-1 = 15

$$S = \begin{bmatrix} 15 & if \quad r \geq T = 8 \\ 0 & Otherwise. \end{bmatrix}$$

Output image A is given by, $A = \begin{bmatrix} 0 & 15 & 0 \\ 15 & 0 & 15 \\ 0 & 15 & 0 \end{bmatrix}$ **ANS**

Q6(b)(2) Intensity Level Slicing Transformation

- Intensity Level Slicing with Background is defined as,

$$S = \begin{cases} L-1 & if \ \ a \le r \le b \\ r & Otherwisse \end{cases}$$

Put L-1 = 15, a=6, and b=12

$$S = \begin{cases} 15 & if \ \ 6 \le r \le 12 \\ r & Otherwisse \end{cases}$$

Output image B is given by, $B = \begin{bmatrix} 2 & 13 & 4 \\ 15 & 15 & 15 \\ 0 & 15 & 3 \end{bmatrix}$ **ANS**

Q6(b)(3) Median Filtering

Output image pixel value at center location is given by

$$C(1,1) = \ Median \begin{bmatrix} 2 & 13 & 4 \\ 15 & 6 & 12 \\ 0 & 9 & 3 \end{bmatrix}$$

C(1,1) = median { 2, 13, 4, 15, 6, 12, 0, 9, 3 }

C(1,1) = median { 0, 2, 3, 4, 6, 9, 12, 13, 15 }

C(1,1) = 6 ANS

Q6(b)(4) Negation Transformation

- It is defined as S=T(r)
  Where T is Digital Negative Tx function
  such that, S = (L-1) - r Put L-1 = 15

By substituting we get, **S = 15 - r**

Output Image D is then given by, $D = \begin{bmatrix} 13 & 2 & 11 \\ 0 & 9 & 3 \\ 15 & 6 & 12 \end{bmatrix}$

Q7(a) Write short note on Animation in Computer Graphics. [7]

Q7(b) Derive mid-point circle algorithm. [8]
