Solution: Statistics and Probability EXAM: MCA SEM – II (CBCGS) DEC – 2016

QP:751002

Q.1

a)

n=100 mean = 40 SD = 10

$\sum xi = 4000$

$\sigma = \surd \sum xi^2 /n - (\sum xi/n)^2$ $\sum xi^2 = 170000$

corrected data : $\sum xi = 4000 - (30+12) + (3+27) = 3928$

$\sum xi^2 = 164654$

corrected mean = 39.28

corrected SD = 10.18

b)

Let missing frequencies be f1 and f2

Expenditure (in Rs.)	No. of Families	c.f
0-20	14	14
20-40	f1	14+f1
40-60	27	41+f1
60-80	f2	41+f1+f2
80-100	15	56+f1+f2
	56+f1+f2	

Total number of families = 100

56+f1+f2 = 100 f1 +f2 = 44………(1)

Median = 50 median class = 40-60 f=27 cf = 14+f1

Median = l1 + (l2-l1)/f * (N/2- cf)

F1-f2 = 1……(2)

f1 = 22.5

f2 = 21.5

c)

n(s) = 36

let A be the event that the number selected is divisible by 3

n(A) = 12 P(A) = 12/36 = 1/3

Let B be the events that the number selected is perfect square

n(B) = 6 P(B) = 6/36 = 1/6

n(AΠ B) = 2 P(A Π B) = 2/36 = 1/18

P(AUB) = 4/9

d)

Expectation theorem

Q.2

a)

If X and Y are two random variables having joint density function

F(x,y) = 2 ; 0 < x < 1 , 0 < y < x

= 0 ; otherwise

		i) Find the marginal density functions of X and Y. Marginal density function of X $F_x(x) = 2x \quad 0<x<1$ $= 0$ otherwise Marginal density function of Y $F_y(y) = 2(1-y) \quad 0<y<x$ $= 0$ otherwise **ii) Find conditional density function of Y given X and X given Y.** The conditional density function of Y given X is 1/x The conditional density function of X given Y is 1/(1-y) iii) Check for independence of X and Y. X and Y are not independent
	b)	**Calculate the Bowley's coefficient of skewness** N = 83 N/4 = 83/4 = 20.7 3N/4 = (3*83)/4 = 62.25 Q1 = 16.48 Q2 = 22.16 Q3 =27.95 **Bowley's coefficient = (Q3 +Q1 – 2*median)/(Q3-Q1)=0.0096**
	c)	**i)** What is the best test score? **Ans: 100** **ii)** How many students took the test? **Ans: 30** **iii)** How many students scored 90? **Ans: 2** **iv)** What is the lowest score? **Ans: 61** **v)** Find the difference between the high and low scores. **Ans: 39**
Q.3	**a)**	**Rank correlation coefficient** Judge 1 -> R1 Judge 2 -> R2 Judge 3 -> R3 $\sum d12^2 = 74$ $\sum d13^2 = 156$ $\sum d23^2 = 44$ $\rho12 = 0.5515$ $\rho13 = 0.0545$ **$\rho23 = 0.7333$** **judge 2 and 3 has the nearest approach to beauty**
	b)	$E(x) = 0.9583$ $E(x^2) = 1.7916$ $E(y) = 0.875$
	c)	Total letters in the word 'failure' are 7 Letters can be arranged in 7! Ways

Let A be the event that consonants may occupy only odd position.
There are 3 consonants in the word failure and 4 odd positions
n(A) = 4*3 *2 *4! = 576
P(A) = 576/7! = **0.1142**

Q.4 a) **Bayes Theorem : Theory**

Let E1, E2, & E3 denote the events that a bolt selected at random is manufactured by machines A, B & C respectively.
Let E denote the event that the bolt is defective.
P(E1) = 0.25, P(E1) = 0.35, P(E1) = 0.40
The Probability that the bolt is defective, given that it is manufactured by A is
P(E|E1) = 0.05
The Probability that the bolt is defective, given that it is manufactured by B is
P(E|E2) = 0.04
The Probability that the bolt is defective, given that it is manufactured by C is
P(E|E3) = 0.02
By using Baye's Theorem,
Probability that randomly selected defective bolt is manufactured by machine A is

$$P(E1|E) = \frac{P(E1)P(E|E1)}{\sum_{i=1}^{3} P(Ei)P(E|Ei)} = \frac{0.25 * 0.05}{0.035} = \frac{0.00125}{0.035} = 0.363 \ OR \frac{25}{69}$$

Probability that randomly selected defective bolt is manufactured by machine B is

$$P(E2|E) = \frac{P(E2)P(E|E2)}{\sum_{i=1}^{3} P(Ei)P(E|Ei)} = \frac{0.35 * 0.05}{0.035} = \frac{0.0014}{0.035} = 0.406 \quad OR \ \frac{28}{69}$$

Probability that randomly selected defective bolt is manufactured by machine C is

$$P(E3|E) = \frac{P(E3)P(E|E3)}{\sum_{i=1}^{3} P(Ei)P(E|Ei)} = \frac{0.40 * 0.02}{0.035} = \frac{0.00140}{0.035} = 0.2330 \ OR \ \frac{16}{69}$$

b) **(AB) = 128 (αB) = 384 (Aβ) = 24 (αβ)= 72**

	A	α	Total
B	128	384	512
β	24	72	96
Total	152	456	608

(A) * (B) / N = (152 * 512)/608 = 128
Since (A) * (B) / N = (AB)
Hence A and B are independent

c) Total frequency of 10 digits is 10,000

Expected frequency = 10000/10 = 1000

Digits	Observed Freq(O)	Expected Freq (E)	$(O-E)^2$	$(O-E)^2/E$
0	1026	1000	676	0.676
1	1107	1000	11449	11.449
2	997	1000	9	0.009
3	966	1000	1156	1.156
4	1075	1000	5625	5.625
5	933	1000	4489	4.489
6	1107	1000	11449	11.449
7	972	1000	784	0.784
8	964	1000	1296	1.296
9	853	1000	21609	21.609
				$\sum$ =58.542

$\chi = \sum((O-E)^2 / E)$

$= 58.542$

Since calculated value is greater than tabulated value(16.92).

The digits 0,1,2,...,9 are not uniformly distributed

Q.5 a) **n=10 mean x = 764.7 mean y = 2.85**

i) byx = 0.00358

Regression Equation of Y on X is $Y - \bar{y} = byx\,(X - \bar{x})$

Regression equation of Y on X is (y – 2.85) = 0.00358(x- 764.7)

Bxy = 251.9587

Regression Equation of X on Y $X - \bar{x} = bxy\,(Y - \bar{y})$

Regression equation of X on Y is (X – 764.7) = 251.9587(y-2.85)

Karl Pearson's Correlation coefficient $r = \pm\sqrt{byx * bxy}$

ii) Karl Pearson's correlation coefficient r = 0.9494

iii) Delivery time in days for 1000 miles

(y – 2.85) = 0.00358 (x – 764.7)

(y – 2.85) = 0.00358 (1000 – 764.7)

Y = 3.6923days

iv) Distance in miles for 2.5 days

(x – 764.7) = 251.9587 (y – 2.85)

(x – 764.7) = 251.9587 (2.5 – 2.85)

X = 676.5144 miles

b) N =150 N/4 = 150/4 = 37.5 3N/4 = 112.5

Q1 = 31.75

Q3 = 62.625

Quartile deviation = (Q3 – Q1)/2 = 15.4375

c) P(P) = 0.8 P(W) = 0.7 P(P Π W) =0.65

i) P(PUW) = 0.85

ii) P(PUW) = P(PUW)’ (Demorgan’s law)

= 0.15

iii) P(P Π W’) = 0.15

Q.6 a) **Box and Whisker diagram**

Given observations in ascending order 1,3,3,6,6,7,7,10

Median =Q2=6

Median of lower quartile = 3

Median of upper quartile = 7

1 is smallest value and 10 is greatest value

By using this draw required diagram

b) **N = 1000 (A) =600 (B) = 500 (AB)= 50**

	A	α	Total
B	50	450	500
β	550	-50	500
Total	600	400	1000

Since (αβ)= -50 the given data is inconsistent.

c) Given n=10, sample mean $\bar{x}$ =0.024, sample standard deviation s=0.002

Null Hypothesis. H_0 : μ =0.025cm, (i.e. there is no significant difference between sample mean $\bar{x}$ =0.024 population mean: μ =0.025)

Alternative Hypothesis. H_1 : μ ≠ 0.025cm

Under H_0 the test statistic is $t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = -1.5 \; or \; (1.5 \; if \; \sqrt{n-1} \; = -3)$

Tabulated t for 9 d.f at 5% LOS is 2.262, |t|=1.5<2.262

calculated value of t $< tabulted \; valueof \; t$ ∴ H_0 is accepted

There is no significant deviation.

d) Value of K = 3/10

Median = (6 - √6)/3 since median lies between 0 and 2