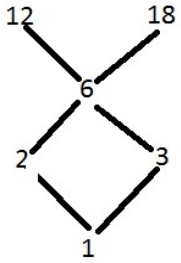
N.B.: (1) Q1. is compulsory, attempt any 3 questions out of remaining six questions
(2) Assume any necessary data to justify the same
(3) Figures to the right indicate full marks
(4) Use of scientific calculator is allowed

		[3 hours] Marks:80	
Q1	a)	If A={1, 2, 3, 6, 12, 18} and the partial order relation R is the divides relation i.e a|b aRb iff (a divides b). i) Draw the Hasse diagram for the poset (A,R) ii) Find the minimal elements, maximal elements, least & greatest elements if exists. iii) If B= {6, 12, 18} and find all the lower bounds and upper bounds of B and LUB and GLB of B	(10)

SOLN

i) Hasse Diagram



ii) Maximal elements are 12,18, Minimal element is 1,
Least element is 1, No greatest element exists

iii) No Upper bounds of B , Lower bounds of B ={1,2,3,6}

No upper bounds of B so no LUB (B), GLB (B)=6

Q1	b)	Without using truth table prove $(P \rightarrow Q) \wedge (R \rightarrow Q) \equiv (P \vee R) \rightarrow Q$	(05)

SOLN

LHS : $(P \rightarrow Q) \wedge (R \rightarrow Q)$

$\equiv (\sim P \vee Q) \wedge (\sim R \vee Q)$

$\equiv (Q \vee \sim P) \wedge (Q \vee \sim R)$

$\equiv Q \vee (\sim P \wedge \sim R)$

$\equiv Q \vee \sim (P \vee R)$

$\equiv \sim (P \vee R) \vee Q$

$\equiv (P \vee R) \rightarrow Q$ $= RHS$

Q1	c)	What are the characteristics of a complex business problem, explain any two	(05)

SOLN

Characteristics Of Complex Business Problems:

- The number of possible solutions is so large that it precludes a complete search for the best answer.
- Problem exists in a time changing environment.
- The problem is heavily constrained.
- There are many (Possibly conflicting) objectives.
- Other characteristics are incomplete information, noisy data and uncertainly.

Any two of above points needs to be explained

Q2 a) The board of directors have to choose a leader for a company whose founder is about to retire. There are three competing candidates TOM, DICK & HARRY and four competing criteria Experience Education, Charisma and Age. Use AHP to choose the most suitable candidate. (15)

(**The CMI, consistency index and consistency Ratio need not be calculated**)

The comparison matrix for pair wise Criteria is given below

CRITERIA	EXPERIENCE	EDUCATION	CHARISMA	AGE
EXPERIENCE	1	4	3	7
EDUCATION	1/4	1	1/3	3
CHARISMA	1/3	3	1	5
AGE	1/7	1/3	1/5	1

Also, the Relative criteria for alternatives is

EXPERIENCE	TOM	DICK	HARRY
TOM	1.00	1/4	4.00
DICK	4.00	1.00	9.00
HARRY	1/4	1/9	1.00

EDUCATION	TOM	DICK	HARRY
TOM	1.00	3	1/5
DICK	1/3	1.00	1/7
HARRY	*5*	7	1.00

CHARISMA	TOM	DICK	HARRY
TOM	1.00	5	9.00
DICK	1/5	1.00	4.00
HARRY	1/9	1/4	1.00

AGE	TOM	DICK	HARRY
TOM	1.00	1/3	5.00
DICK	3.00	1.00	9.00
HARRY	1/5	1/9	1.00

SOLN

Step1) **Find weights for Relative (Criteria Vs Criteria)**

	EXPERIENCE	EDUCATION	CHARISMA	AGE
EXPERIENCE	1	4	3	7
EDUCATION	1/4	1	1/3	3
CHARISMA	1/3	3	1	5
AGE	1/7	1/3	1/5	1
sum	1.726	8.333	4.533	16

Divide every element by column sum and then take row average

	EXPERIENCE	EDUCATION	CHARISMA	AGE	Row average wt or eigen v
EXPERIENCE	0.579	0.48	0.662	0.438	**0.54**
EDUCATION	0.145	0.12	0.074	0.188	**0.132**
CHARISMA	0.193	0.36	0.221	0.313	**0.272**
AGE	0.083	0.04	0.044	0.063	**0.058**

Step2) Find Weights of each of the Critera (Alternative Vs Alternative)

EXPERIENCE	TOM	DICK	HARRY
TOM	1	1/4	4
DICK	4	1	9
HARRY	1/4	1/9	1
sum	5.25	1.361	14

Divide every ele by column sum & then take row avg

EXPERIENCE	TOM	DICK	HARRY	Row average wt or eigen v
TOM	0.19	0.184	0.286	**0.22**
DICK	0.762	0.735	0.643	**0.713**
HARRY	0.048	0.082	0.071	**0.067**

EDUCATION	TOM	DICK	HARRY
TOM	1	3	1/5
DICK	1/3	1	1/7
HARRY	5	7	1
sum	6.333	11	1.343

Divide every ele by column sum & then take row avg

EDUCATION	TOM	DICK	HARRY	Row average wt or eigen v
TOM	0.158	0.273	0.149	**0.193**
DICK	0.053	0.091	0.106	**0.083**
HARRY	0.79	0.636	0.745	**0.724**

CHARISMA	TOM	DICK	HARRY
TOM	1	5	9
DICK	1/5	1	4
HARRY	1/9	1/4	1
sum	1.311	6.25	14

Divide every ele by column sum & then take row avg

CHARISMA	TOM	DICK	HARRY	Row average wt or eigen v
TOM	0.763	0.8	0.643	**0.735**
DICK	0.153	0.16	0.286	**0.2**
HARRY	0.085	0.04	0.071	**0.065**

AGE	TOM	DICK	HARRY
TOM	1	1/3	5
DICK	3	1	9
HARRY	1/5	1/9	1
sum	4.2	1.444	15

Divide every ele by column sum & then take row avg

AGE	TOM	DICK	HARRY	Row average wt or eigen v
TOM	0.238	0.231	0.333	**0.267**
DICK	0.714	0.693	0.6	**0.669**
HARRY	0.048	0.077	0.067	**0.064**

Step3)

The composite impact table

WEIGHTS	0.54	0.132	0.272	0.058
Criteria -->	EXPERIENCE	EDUCATION	CHARISMA	AGE
TOM	0.22	0.193	0.735	0.267
DICK	0.713	0.083	0.2	0.669
HARRY	0.067	0.723	0.065	0.064

Composite impact of TOM = 0.359,
Composite impact of DICK = **0.489**,
Composite impact of HARRY= 0.153.
Best composite score is : 0.489, Best Alternative is to choose **DICK**

Q2	b)	Use Mathematical induction to prove the property P(n) P(n): 3^n+2n-1 is divisible by 4 $\forall$ n $\in$ N	(05)

SOLN

P(1) is true, Assume P(k) is true => 3^k+2k-1

Claim : P(k+1) is true i.e. $3^{k+1}+2(k+1)-1$ is divisible by 4 , 3^k+2k-1 is divisible by 4

Further solving we get $3^{k+1}+2(k+1)-1 = 4m$ for some integer m

=RHS. Hence proved

Q3 a) Use SAW method to determine the best car. The beneficiary criteria are Durability in years and Resale value , others are non beneficiary criteria (10)

The measures for different criteria are given in the table below

Type of Car	MAINTANCE COST in Rs.	Purchase PRICE IN Rs.	DURABILITY IN YEARS	RESALE VALUE in Rs.
CAR1	800	350000	6.5	100000
CAR2	1000	1000000	10	450000
CAR3	1250	650000	10	290000

The weights for different criteria are

Type of Car	MAINTANCE COST in Rs.	Purchase PRICE IN Rs.	DURABILITY IN YEARS	RESALE VALUE in Rs.
Weight	0.15	0.4	0.25	0.2

SOLN

Sum of Weights is 1 , Already Normalized

As Durability in years and Resale value are Beneficiary and

Maintenance cost and Resale Value are Non beneficiary, we need to normalize the measures

Weight	0.15	0.4	0.25	0.2
Cri_type	-	-	+	+
	MAINTANCE COST in Rs.	Purchase PRICE IN Rs.	DURABILITY IN YEARS	RESALE VALUE in Rs.
CAR1	800	350000	6.5	100000
CAR2	1000	1000000	10	450000
CAR3	1250	650000	10	290000

After Normalizing,

Weight	0.15	0.4	0.25	0.2
	MAINTANCE COST in Rs.	Purchase PRICE IN Rs.	DURABILITY IN YEARS	RESALE VALUE in Rs.
CAR1	1	1	0.65	0.222
CAR2	0.8	0.35	1	1
CAR3	0.64	0.538	1	0.644

Performance scores are

P(CAR1)= **0.76**, P(CAR2)= 0.71, P(CAR3)= 0.69

THE DECISION IS TO CHOOSE THE **CAR1**

Q3	b)	In a screening test for a disease. The frequency of the disease in a population is 0.5%. The test is highly accurate with 5% false positive rate and 10% false negative rate. A person takes the test and it comes positive. Construct a decision tree and use Baye's theorem to determine the probability that he has a disease?	(10)

SOLN

DEFINE THE EVENTS

D+ : The person has a disease, D - : The person does not have a disease

T+ : The test is positive, T - : the test is negative

Given: P(D+)=0.5 % = 0.005, P(D-)=0.995

P(false positive) is probability that (Test is positive, given the person does not have disease)

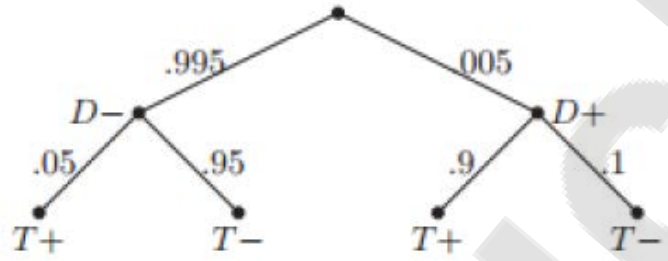
P(false positive) =P(T+|D-)=0.05, P(false negative)=P(T-|D+)=0.1

The Complimentary probabilities are

P(true negative)= P(T-|D-) = 1- P(T+|D-) = 1-0.05 = 0.95

P(true positive)= P(T-|D-) =1- P(T-|D+)=1-0.1=0.9

Decision Tree:



We have to find $P(D+|T+)$

Using Bayes Theorem,

$$We\ have\ to\ find\ P(D+|T+) = \frac{P(T+|D+)\,P(D+)}{P(T+)} \quad \text{... Eq1}$$

P(T+)=0.05 X 0.995 + 0.9 X 0.005 =0.05425

P(T+|D+)=0.9, P(D+)=0.005

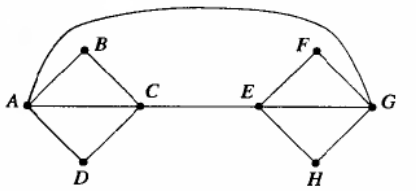
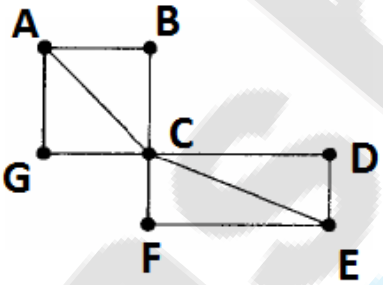
By Eq1 $P(D+|T+) = \frac{(0.9\ X\ 0.005)}{0.05425} = 0.082949$

Q4	a)	State the 'Tower's of Hanoi' problem and obtain the recurrence relation for the same	(10)
SOLN		theory	
Q4	b)	Write the truth table for α and find the principal CNF & Principal DNF α α =(~P V ~Q)→ (P ↔ Q)	(10)

SOLN

1	2	3	4	5	6	7
P	Q	~ P	~ Q	~ PV~ Q	P ↔ Q	(~P V ~Q)→ (P ↔ Q)
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Principal DNF is (P∧Q)V(~P∧~Q), Principal CNF is (~PVQ) ∧(PV~Q)

Q5	a)	The solution of the Recurrence relation $C_0a_n + C_1a_{n-1}+C_2a_{n-2} = f(n)$ is $2^n + 3^n + 5$, where $f(n) = 40$, find C_0, C_1, C_2	(10)
SOLN		$C_0 = 4, \quad C_1 = -20, \quad C_2 = 24$	
Q5	b)	Find the Euler Path and Euler Circuit in the following graphs if they exists  GRAPH G1  GRAPH G2	(10)
SOLN		One of the Euler circuit : for G1 $\pi: A, B, C, A, D, C, E, G, F, E, H. G, A$ One of the Euler path : for G2 *Π: A, C, E, D, C, B, A, G, C ,F, E,* No Euler circuit for G2	
Q6	a)	A={a ,b, c, d, e}, R={(a, a), (a, d),(b, b),(c, d), (c, e),(d, a),(e, b),(e, e)} Determine the R∞ relation using Warshall's Algorithm.	(05)

SOLN Final Warshall Matrix

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

R∞={ (a, a), (a, d),(b, b), (c, a), (c, b), (c, d), (c, e),(d, a), (d, d) , (e, b),(e, e)}

Q6	b)	Determine whether the relation R on the set A is reflective, irreflective, symmetric, asymmetric, antisymmetric, transitive, identity relation. Give the necessary explanation to your answer. A= Set of Real numbers and aRb iff |a-b|≤2	(05)

SOLN

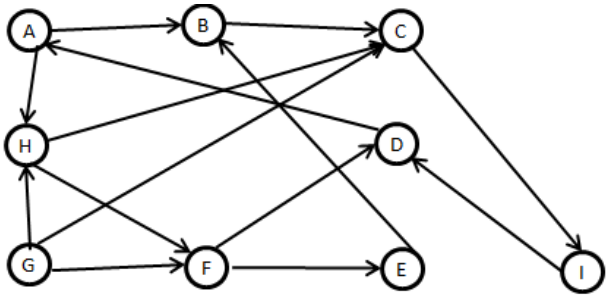
R is reflective, not irreflective,

R is symmetric, not asymmetric, not antisymmetric

R is not transitive, not identity

Q6	c)	Find the particular solution of the recurrence relation $a_n-2a_{n-1}=3x2^n$	(05)
SOLN		$a_n^{(p)}=3n \times 2^n$	

Q6 d) Find the Adjacency Matrix and Adjacency list for the following graph Vertices are shown in circles (05)



SOLN

Adjacency matrix

	A	B	C	D	E	F	G	H	I
A	0	1	0	0	0	0	0	1	0
B	0	0	1	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	1
D	1	0	0	0	0	0	0	0	0
E	0	1	0	0	0	0	0	0	0
F	0	0	0	1	1	0	0	0	0
G	0	0	1	0	0	1	0	1	0
H	0	0	1	0	0	1	0	0	0
I	0	0	0	1	0	0	0	0	0

Adjacency List

A→B→H
B→C
C→I
D→A
E→B
F→D→E
G→C→F→H
H→C→F
I→D